If you want to smoke, don't buy cigarettes: near-anankastics, contexts, and hyper modality

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Abstract

In this discussion paper we are interested in anankastic conditionals such as "if you want to smoke you must buy cigarettes" and near-anankastic conditionals such as "if you want to smoke, you must not buy cigarettes." First, we discuss challenges to representing such conditionals in deontic logic, in particular in relation to the use of context. We do this through a discussion of the Tobacco shop scenario, an example dealing with ambiguity of certain deontic conditionals. Second, we illustrate how ambiguity of natural language can be formally represented through the use of hyper-modalities, using a minimal modal logic for (near-)anankastic conditionals. We illustrate how the hyper-modal setting can disambiguate such conditionals. As the Tobacco shop scenario suggests, in our formalism interaction between antecedent, consequent, and context can reduce ambiguity in the involved conditionals.

Keywords: anankastic conditionals, deontic logic, desire modalities, hyper modalities

1 Introduction

Natural language offers a wide assortment of sometimes ambiguous deontic expressions. For example, consider the sentence "if you want to smoke, you must buy cigarettes." This natural language sentence can be interpreted in at least two ways. On the one hand, we may say that the best means to smoke is to buy cigarettes. On the other hand, we may say that the most ideal way to satisfy your desire is to buy cigarettes (better than, say, stealing cigarettes). The former is a teleological interpretation of 'must' (i.e., referring to a goal), and the latter is called a deontic interpretation (i.e., referring to a duty). A modality such as 'must' has many different interpretations [9,15]. As

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a basic example, the utterance 'it must rain' may refer to an epistemic necessity which says that it cannot but rain, but it could also refer to an optimal ideal expressing that it ought to rain. Often, context may help to disambiguate. For instance, if it does not rain here and now, then we know 'must' cannot receive an epistemic necessity reading. Hence, different contexts will imply different logical reasoning with the same modality. For example, epistemic necessity may be an $\mathsf{S5}$ modality, whereas deontic obligation is a KD modality. Likewise, the different interpretations of the conditional "if you want to smoke, you must buy cigarettes" will have distinct logical formalisations. Following [7], we call modalities that may receive different interpretations in different contexts *hyper-modalities*.

In this paper, we discuss hyper-modality through a discussion of a challenging example concerning *anankastic* and *near-anankastic* conditionals: the Tobacco shop scenario. The anankastic conditional "if you want to smoke, you must buy cigarettes" is one of the scenario's central premises. Von Wright is said to be the first to adopt the term 'anankastic' in his philosophy of agency [15], but thorough investigation of such conditionals began with the work of Sæbø [12]. In [12], Sæbø points out that the nature of 'desire' in the antecedent generates some challenges when it comes to the interpretation of the modal 'must' in the consequent, challenges which are particular to anankastic conditionals. Since then, alternative accounts have been proposed (e.g., [4,13,14]), properly introducing anankastics to the research agenda of deontic modality.

The contribution of this discussion paper is twofold. First, we argue that the analysis of deontic modalities—such as 'must'—in natural language expressions can bring new challenges to deontic logic. In particular, we discuss how the consequent plays an important role in evaluating an anankastic conditional, via interaction with the conditional's embedded context and antecedent. Second, we argue that logical techniques can help to bring some aspects traditionally referred to pragmatics, within the reach of logical analysis. That is, we develop a hyper-modal setting in which ambiguous (near-)anankastic conditionals—such as those in the Tobacco shop scenario—can be formally represented and which facilitates partial disambiguation of such conditionals.

The paper is structured as follows: In Section. 2, we discuss the Tobacco shop scenario and (near-)anankastic conditionals. Section. 3 contains a modal logic for the Condoravdi-Lauer analysis of (near-)anankastics and we formalise four types of such conditionals. Section. 4 extends this logic to the hyper-modal setting, internalising part of the pragmatics of interpreting (near-)anankastics. In Section. 5, we provide a hyper-modal analysis of the Tobacco shop scenario.

2 The Tobacco shop scenario

The development of deontic logic has been driven by deontic benchmark examples [11]. In this paper we are interested in the *Tobacco shop scenario*, a scenario which circulates since at least 2016 in various forms [5]:

Dr. Smoke wanders through the university's inner courtyard. Prof. Prag-

> matics notices a slight disturbance in Dr. Smoke's mood. She asks him: "what's on your mind?" Smoke shares with her his craving for a cigarette. Prof. Pragmatics replies "if that is so, then you must go to the tobacco shop!" At that moment, Prof. Restraint crosses the lawn and, by chance, catches Pragmatics' last remark. Restraint asks: "what is going on here?" Pragmatics: "He wants a cigarette!" Prof. Restraint looks surprised: "if that is the case," she exclaims, "then you surely should not go to the tobacco shop. "

The Tobacco shop scenario illustrates a scenario in which Dr. Smoke (henceforth, S) receives seemingly incompatible advice, conditional on his desire to smoke. At face value, we have two similar conditional premises (henceforth, P1 and P2) which share the same antecedent:

If S wants to smoke, then S must buy cigarettes. (P1)

If S wants to smoke, then S must not buy cigarettes. (P2)

Provided that the conditionals involved are of the same form ([2] makes a strong case for uniformity of conditionals), we have to accept the following inference:

If S wants to smoke, then S must buy cigarettes and S must not buy cigarettes. (1)

In any deontic logic that allows for factual detachment and monotonicity of its modal operators, while also adopting a deontic consistency axiom [8] a logical inconsistency of P1 and P2 arises in the light of S's actual desire:

S wants to smoke. (2)

Which gives us:

S must buy and not buy cigarettes. (3)

A consequence such as (3) is not just undesirable, it also does not seem to do justice to the nature of the involved conditionals. Premises P1 and P2 do not merely express conflicting obligations given a shared antecedent, they convey additional information: the relation between smoking and buying cigarettes is clearly of a different nature than the relation between smoking and not buying cigarettes. For instance, by looking at the antecedent and consequent of P1, we observe that buying cigarettes, as an activity, may serve as a means for smoking. This is not the case for P2. In fact, not buying may even prove obstructive to satisfying one's desire to smoke. We find that the interpretation of 'must' in P2 differs from the one adopted in P1: 'must' is a hyper-modality. In the case of P2, the consequent suggests the need for additional context in which the conditional must be embedded. Conditionals that relate statements of desire to statements of must, are called *(near-)anankastic conditionals*.

Many of the benchmark examples in deontic logic revolve around challenges of reasoning with conditionals in normative settings [8]. Likewise, we find that the Tobacco shop scenario focuses on a specific, yet ubiquitous, type of conditionals: anankastic and near-anankastic conditionals. In this section we will see that, whereas often only the antecedent is considered in evaluating the consequent [1], the consequent of a (near-)anankastic conditional plays an important role in evaluating the conditional, through interacting with the conditional's embedded context and antecedent.

2.1 Terminology

We first go through some terminological matters. The group of conditionals we are interested in share the following structure: 'if you want ϕ, you must ψ'. Depending on the disambiguation of the modalities 'desire' and 'must', but also on the relation between their internal structure ϕ and ψ, we may obtain different conditionals called *anankastic* and *near-anankastic* conditionals. We recall the terminology of Condoravdi and Lauer [4]:

$$\text{If } \overbrace{S \quad \underbrace{\text{wants to}}_{\text{desire predicate}} \quad \underbrace{[S]\text{ smoke}}_{\text{internal antec.}}}^{\text{antecedent}}, \text{ then } \overbrace{S \quad \underbrace{\text{must}}_{\text{modal}} \quad \underbrace{[S]\text{ buy cigarettes}}_{\text{prejacent}}}^{\text{consequent}}$$

An anankastic conditional transmits that the complement of 'must' functions as a necessary *precondition* for the realisation of the complement of 'desire'. See [4] for a discussion. A near-anankastic conditional, on the other hand, has the same general structure but lacks the relation between internal antecedent and consequent as one of necessary precondition.

Given the above distinction, we find that the two central premises $\mathsf{P1}$ and $\mathsf{P2}$ of the Tobacco shop scenario are, respectively, an anankastic and a near-anankastic conditional. Namely, $\mathsf{P1}$ expresses a positive relation between the internal antecedent 'S smokes' and the prejacent 'S buys cigarettes', i.e., S's buying cigarettes is instrumental to S's smoking. In this particular case, the relation is a *best-means* relation which indicates that buying is an optimal means serving the goal of smoking. When 'must' is taken to refer to an optimal realisation of a goal, we say it receives a *teleological* reading. Premise $\mathsf{P2}$ does not express such a relation (in fact, it hints at a relation to the contrary) and for that reason it is called a near-anankastic conditional. As a first observation we find that, in order to determine the nature of the conditional we must thus go into its *substructure*: to correctly interpret the conditional, we must (i) determine the relation between the four central components of a (near-)anankastic conditional and (ii) disambiguate the involved modalities 'must' and 'desire'. Interpreting the substructure of the conditional, subsequently, often depends on the context in which the statement occurs.

2.2 Two types of desire and two types of obligations

The right interpretation of the 'desire' expressed in a (near-)anankastic's antecedent, plays a central role in correctly interpreting the consequent (and vice versa). Condoravdi and Lauer [4] distinguish between two types of desires: *mere desires* and *action-relevant desires*. Mere desires are desires that are 'psychological facts' (nothing more), whereas action-relevant desires reflect the agent's goal and a corresponding intention to realise that goal. We refer to D^1 and D^2 as mere desires, respectively, action-relevant desires. Since the latter is related to action (i.e., a goal), it is subject to additional constraints. Hence, the two notions have a different logic which will influence the logical behaviour of conditionals in which they occur. In other words, 'desire' in a (near-)anankastic conditional is a *hyper modality* too. Disambiguation may give:

$$D^2 \text{ smoke} \Rightarrow O \text{ buy cigarettes.}^2 \tag{p1}$$

$$D^1 \text{ smoke} \Rightarrow O \text{ not buy cigarettes.} \tag{p2}$$

Similarly, we can distinguish different kinds of obligation. For instance, O^1 may denote a teleological 'must', whereas O^2 represents a deontic 'must':

$$D \text{ smoke} \Rightarrow O^1 \text{ buy cigarettes.} \tag{p1}$$

$$D \text{ smoke} \Rightarrow O^2 \text{ not buy cigarettes.} \tag{p2}$$

Given these possible readings of 'desire' and 'must' we already obtain four different interpretations of P1 and P2. We come back to this in Section. 3.

2.3 Various deontic contexts

We represent the *context* of a sentence Pi by Δ_i. The context expresses the conversational background in the light of which a sentence is uttered (cf. [9]). Such a context may contain facts, beliefs, desires, obligations, and what have you (from an agentive perspective we may call the context epistemic, in the sense that it expresses that which is *known* to the speaker of the sentence). Thus, we may take P1 and P2 as implicit renditions of:

$$\Delta_1 \wedge D \text{ smoke} \Rightarrow O \text{ buy cigarettes.} \tag{p1}$$

$$\Delta_2 \wedge D \text{ smoke} \Rightarrow O \text{ not buy cigarettes.} \tag{p2}$$

Contexts may change how we interpret the two conditionals and their involved modalities. In multi-agent scenarios, in which utterances come from different speakers, different contexts for the individual sentences are likely to occur. For instance, in the Tobacco shop scenario Prof. Restraint may know of Dr. Smoke's desire (or promise) to stop smoking, whereas Prof. Pragmatics does not. Usually, when the relevant context merely contains facts and common knowledge, it is left out of the conditional. Think of a case in which you need to apologise because you did not keep a promise. If it is common knowledge that "you must keep a promise," the conditional may be safely abbreviated to "if you break a promise, you should apologise." Unfortunately, often such common knowledge is falsely assumed and this may lead to ambiguity and miscommunication. In such cases, just as in the Tobacco shop scenario, we must ask for certain context to be made explicit. For instance, upon inquiry Prof. Pragmatics may recall that the tobacco shop is just around the corner, thus making the attainment of Smoke's goal most optimal. In other cases, looking at the content of a conditional's constituents, may help to reconstruct possible contexts and interpretations.

To illustrate the above, Prof. Restraint may recall that Dr. Smoke also has

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[2] In what follows, we write p1 and p2 to indicate alternative formal readings of P1 and P2.

a desire to be healthy, which would be unattainable in the light of smoking:

$$D \text{ smoke} \Rightarrow O \text{ buy cigarettes.} \qquad \text{(p1)}$$

$$D \text{ healthy} \wedge D \text{ smoke} \Rightarrow O \text{ not buy cigarettes.} \qquad \text{(p2)}$$

Given the additional context, the consequent in p2 above seems to suggest a priority for health over the desire to smoke. Here, the consequent provides additional information about the context too. If Dr. Smoke would buy cigarettes he would, given his desire to smoke, most likely start to smoke, thus compromising his health. The consequent seems to suggest that (i) the context contains an action-relevant desire and (ii) the antecedent contains a mere desire. Other contexts worth investigating are factual, normative, and intentional contexts.

2.4 Analysis: four observations concerning (near-)anankastics

In conditional logic, it is normally assumed that the context of a conditional is determined by the antecedent only [1]. One of the interesting aspects of the Tobacco shop scenario, is that this approach is no longer sufficient: P1 and P2 have the same antecedent but different consequents. In this analysis, interpreting conditionals such as P1 and P2 depends on the (mutual) interaction between antecedent, consequent, and context. Ambiguous sentences such as P1 and P2 may receive their correct interpretation through this interaction and additional context. Through disambiguation, the antecedents of the two conditional obligations may no longer be the same. Thus, we find that simple applications of aggregation to P1 and P2 are not always warranted for (cf. inference (1)) and consequently the pair of sentences is no longer inconsistent in standard deontic logic (cf. inference (3)).

In what follows, we adopt an explicit context to specify and investigate possible interactions between and interpretations of desire and obligation modalities. We have the following *general* representation of the Tobacco shop scenario:

$$(\Delta_1, D \text{ smoke}) \Rightarrow O \text{ buy cigarettes.} \qquad \text{(p1)}$$

$$(\Delta_2, D \text{ smoke}) \Rightarrow O \text{ not buy cigarettes.} \qquad \text{(p2)}$$

Still, the way in which we interpret these conditionals depends on the possible interpretations that can be assigned to the ambiguous modalities 'desire' and 'must'. Namely, in the above D and O represent hyper-modalities that may receive different logical interpretations depending on their appearance in the conditional (together with their corresponding context Δ_i). Several readings of D and O are possible, but in the present work we limit their possible interpretations to those discussed above, i.e., D^1, D^2, O^1, O^2.

We make four key observations about the logic of (near-)anankastics:

Role of consequent The consequents of P1 and P2 must inform us on their relation with their respective antecedents and contexts.

Ambiguity of modalities The semantic interpretations of 'desire' and 'must' may vary from context to context. The constituents of the conditional, together with its context, must aid in determining the appropriate interpretations of these ambiguous modalities.

Ambiguity of conditionals The semantic interpretation of a conditional, such as P1 and P2, will likewise vary with its context. This depends partially on resolving ambiguity of the modalities 'desire' and 'must'.

Aggregation rule The different nature of P1 and P2 suggests that the application of aggregation to P1 and P2 may not be warranted for.

Formal analysis of the Tobacco shop scenario must thus explain how the antecedent *together with* the consequent receive their interpretation. An immediate question would be: *can we still formally reason with (near-)anankastic conditionals, even if we cannot completely resolve ambiguity?* We come back to this in Section. 5 where we formally discuss the Tobacco shop scenario.

3 Condoravdi-Lauer and (near-)Anankastic conditionals

Anankastic conditionals have been extensively discussed in formal linguistics. Many authors developed non-standard ways to deal with the relation between desires and obligations in anankastic conditionals [9,13,14,4]. For example, in the setting of Kratzer [9], the obligations are based on a so-called ordering source, and this ordering source is updated by the desire in the antecedent (i.e., the restrictor analysis). Condoravdi and Lauer [4] provide an account that does not only address the compositionality problems that arises in previous approaches, their account is generalised to the inclusion of near-anankastic conditionals (dealing with several natural language examples of (near-)anankastics which previous accounts could not satisfactorily address). They argue that better results can be obtained by adopting a standard approach with counterfactual implications and standard dyadic, teleological obligations. Without taking a stance in this debate, since the Tobacco shop scenario deals with both types of conditionals, we will base our logic on the Condoravdi-Lauer approach [4]. In this section, we present the modal logic La, which allows us to formally represent four different interpretations of (near-)anankastic conditionals. In Section. 4, we will extend this account to a hyper-modality setting in order to reason with ambiguity in such conditionals.

3.1 A Modal logic for Anankastic Conditionals

The properties of desire and teleological modalities are taken from Condoravdi and Lauer [4], and we refer to their paper for an in-depth discussion of these properties and the various alternatives. The two kinds of desire, D^1 and D^2—i.e., mere-desire, respectively action-relevant desire—are differentiated by the property 'conjunction introduction' and a 'consistency' requirements (both are not valid for D^1, but are valid for D^2). For our purposes, we make a few modifications and simplifications: The teleological modality $O^1(\phi, \psi)$ reads 'ϕ holds when ψ is optimally realised'. We adopt a dyadic, deontic obligation $O^2(\phi, \psi)$ which reads 'ϕ is obligatory given ψ' and adopt a triadic conditional $(\phi, \psi) \Rightarrow \theta$ expressing "given context ϕ, if ψ, then θ".[3] See [6] for motivating

[3] Another way to look at $(\phi, \psi) \Rightarrow \theta$ is to take $\Rightarrow$ as a stereotypicality conditional, in line with the covert outer modal in [4]: 'in the most stereotypical ϕ and ψ worlds, θ holds'.

the use of ternary conditionals. A universal modality $\Box$ is used to represent facts rigid across contexts: $\Box\phi$ reads 'ϕ holds universally'. We do not formalise desires as priority rankings. The language $\mathcal{L}_{\mathsf{La}}$ is defined by the following BNF:

$$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid D^1\phi \mid D^2\phi \mid O^1(\phi,\phi) \mid O^2(\phi,\phi) \mid (\phi,\phi) \Rightarrow \phi \mid \Box\phi$$

with $p \in \mathsf{Atoms}$. The connectives $\neg$ and $\vee$ are read as usual, and other connectives are obtained in the standard way. We use $\Diamond$ for the dual of $\Box$. The modalities are interpreted as discussed above.

We provide a Hilbert-style axiomatization for the logic of anankastics La.

Definition 3.1 The *Logic of anankastics* La extends the logic S5 for $\Box$ with:

A1. $(D^2\phi \wedge D^2\psi) \to D^2(\phi \wedge \psi)$ (C)

A2. $\mathsf{X}(\psi \wedge \theta, \phi) \to (\mathsf{X}(\psi, \phi) \wedge \mathsf{X}(\theta, \phi))$ for $\mathsf{X} \in \{O^1, O^2\}$ (M)

A3. $\mathsf{X}(\psi, \phi) \wedge \mathsf{X}(\theta, \phi)) \to \mathsf{X}(\psi \wedge \theta, \phi)$ for $\mathsf{X} \in \{O^1, O^2\}$ (C)

A4. $\neg D^2\bot$ (P)

A5. $\neg(D^2\phi \wedge D^2\neg\phi)$ (D)

A6. $\Diamond\phi \to \neg(\mathsf{X}(\psi, \phi) \wedge \mathsf{X}(\neg\psi, \phi))$ for $\mathsf{X} \in \{O^1, O^2\}$ (D)

A7. $\Diamond\phi \to \mathsf{X}(\phi, \phi)$ for $\mathsf{X} \in \{O^1, O^2\}$ (Id)

A8. $\neg\mathsf{X}(\phi, \bot)$ for $\mathsf{X} \in \{O^1, O^2\}$ (F)

A9. $\Diamond(\phi \wedge \psi) \to (\phi, \psi) \Rightarrow (\phi \wedge \psi)$ (Id)

A10. $(\phi, \psi) \Rightarrow \theta \to (\Diamond(\phi \wedge \psi) \wedge \Diamond\theta)$ (F)

A11. $(\phi, \psi) \Rightarrow (\chi \wedge \theta) \to ((\phi, \psi) \Rightarrow \chi \wedge (\phi, \psi) \Rightarrow \theta)$ (M)

A12. $((\phi, \psi) \Rightarrow \chi \wedge (\phi, \psi) \Rightarrow \theta) \Rightarrow (\phi, \psi) \Rightarrow (\chi \wedge \theta)$ (C)

R1. Congruence rule: holds for D^1, D^2, O^1, O^2, and $\Rightarrow$ (all arguments) (RE)

La-derivability and La-theorems are defined as usual [3].

In Def. 3.1, M and C denote monotonicity, respectively, conjunction introduction. P and D are consistency constraints on D^2, O^1, O^2. Id is identity for consistent formulae. F states that no obligation O^1, O^2 holds given $\bot$, and that the antecedent and consequent of $\Rightarrow$ are jointly consistent, respectively, consistent. Since La is a non-normal logic, we use neighbourhood semantics [3]:

Definition 3.2 An La-frame is a tuple $F = \langle W, \mathcal{N}_{D^1}, \mathcal{N}_{D^2}, \mathcal{N}_{O^1}, \mathcal{N}_{O^2}, \mathcal{N}_{\Rightarrow} \rangle$, where $W \neq \emptyset$ is a non-empty set of worlds $w, v, u, ...$ (possibly indexed) and $\mathcal{N}_i$ ($i \in \{D^1, D^2, O^1, O^2, \Rightarrow\}$) are neighbourhood functions such that:

- $\mathcal{N}_j : W \mapsto \mathcal{P}(\mathcal{P}(W))$ for $j \in \{D^1, D^2\}$
- $\mathcal{N}_k : W \mapsto \mathcal{P}(\mathcal{P}(W) \times \mathcal{P}(W))$ for $k \in \{O^1, O^2\}$
- $\mathcal{N}_{\Rightarrow} : W \mapsto \mathcal{P}(\mathcal{P}(W) \times \mathcal{P}(W) \times \mathcal{P}(W))$

F satisfies the following constraints, for all $w \in W$, and all $X, Y, Z, U \subseteq W$:

(c_1) if $Z \in \mathcal{N}_{D^2}(w)$ and $Y \in \mathcal{N}_{D^2}(w)$, then $Z \cap Y \in \mathcal{N}_{D^2}(w)$;

(m_2) $i \in \{O^1, O^2\}$, $(X \cap Y, Z) \in \mathcal{N}_i(w)$ implies $(Y,Z) \in \mathcal{N}_i(w)$ and $(X, Z) \in \mathcal{N}_i(w)$;

(c_2) $i \in \{O^1, O^2\}$, $(X, Z) \in \mathcal{N}_i(w)$ and $(Y, Z) \in \mathcal{N}_i(w)$ implies $(X \cap Y, Z) \in \mathcal{N}_i(w)$;

(p) $\emptyset \notin \mathcal{N}_{D^2}(w)$;

(d_1) if $X \in \mathcal{N}_{D^2}(w)$, then $\overline{X} \notin \mathcal{N}_{D^2}(w)$;

(d_2) $i \in \{O^1, O^2\}$, if $X \neq \emptyset$, $(Y, X) \in \mathcal{N}_i(w)$, then $(\overline{Y}, X) \notin \mathcal{N}_i(w)$;

(id_1) $i \in \{O^1, O^2\}$, if $X \neq \emptyset$, then $(X, X) \in \mathcal{N}_i(w)$;

(f_1) $i \in \{O^1, O^2\}$, if $X = \emptyset$, then $(Y, X) \notin \mathcal{N}_i(w)$;

(id_2) if $X \cap Y \neq \emptyset$, then $(X, Y, X \cap Y) \in \mathcal{N}_{\Rightarrow}(w)$;

(f_2) if $X \cap Y = \emptyset$ or $Z = \emptyset$, then $(X, Y, Z) \notin \mathcal{N}_{\Rightarrow}(w)$;

(m_3) if $(X, Y, Z \cap U) \in \mathcal{N}_{\Rightarrow}(w)$, then $(X, Y, Z) \in \mathcal{N}_{\Rightarrow}(w)$ and $(X, Y, U) \in \mathcal{N}_{\Rightarrow}(w)$;

(c_3) if $(X, Y, Z) \in \mathcal{N}_{\Rightarrow}(w)$ and $(X, Y, U) \in \mathcal{N}_{\Rightarrow}(w)$, then $(X, Y, Z \cap U) \in \mathcal{N}_{\Rightarrow}(w)$;

An La-model is a tuple $M = \langle F, V \rangle$ s.t. F is an La-frame and V is a valuation function assigning atoms $p \in \mathsf{Atoms}$ to sets of worlds; i.e. $V : \mathsf{Atoms} \mapsto \mathcal{P}(W)$.

Last, we semantically evaluate formulae of $\mathcal{L}_{\mathsf{La}}$ as usual:

Definition 3.3 Let M be an La-model, $w \in W$ and $\|\phi\| = \{v \in W | M, v \models \phi\}$:

- $M, w \models p$ iff $w \in V(p)$.
- $M, w \models \neg\phi$ iff $M, w \not\models \phi$
- $M, w \models \phi \vee \psi$ iff $M, w \models \phi$ or $M, w \models \psi$
- $M, w \models \mathsf{X}\phi$ iff $\|\phi\| \in \mathcal{N}_{\mathsf{X}}(w)$ with $\mathsf{X} \in \{D^1, D^2\}$
- $M, w \models \mathsf{Y}(\phi, \psi)$ iff $(\|\phi\|, \|\psi\|) \in \mathcal{N}_{\mathsf{Y}}(w)$ with $\mathsf{Y} \in \{O^1, O^2\}$.
- $M, w \models (\phi, \psi) \Rightarrow \theta$ iff $(\|\phi\|, \|\psi\|, \|\theta\|) \in \mathcal{N}_{\Rightarrow}(w)$.
- $M, w \models \Box\phi$ iff for all $v \in W$, $M, v \models \phi$

Satisfiability, validity, and model-validity are defined as usual [3]. (nb. The operator $\Box$ expresses model-validity.)

Comparing the axioms of Def.3.1 with the properties of Def.3.2, we see that La is highly modular. Consequently, completeness is obtained following the standard approach for neighbourhood semantics [3] (proofs are omitted):

Theorem 3.4 *(SOUNDNESS AND COMPLETENESS) Let $\phi \in \mathcal{L}_{\mathsf{La}}$, and let C_{La} be the class of La-frames: $C_{\mathsf{La}} \models \phi$ iff $\vdash_{\mathsf{La}} \phi$.*

3.2 Anankastics and near-anankastics in the logic La

In this section we discuss formalisations of four teleological (near-)anankastic conditionals. We focus on those that play a role in the Tobacco shop scenario.

The anankastic conditional. Anakastic conditionals are identified by the fact that both the antecedent and consequent receive an action-relevant reading of desire [4]. Let $\Delta \subseteq \mathcal{L}_{\mathsf{La}}$ be the context representing the finite knowledge base of the speaker. The formalised anankastic conditional $(\Delta, D^2\phi) \Rightarrow_{ac} O^1\psi$ is informally interpreted as: "(i) all the most stereotypical worlds consistent with Δ in which $D^2\phi$ holds, are such that whenever all the addressee's known goals, including ϕ, are optimally realised, then ψ holds and (ii) the hypothesised goal ϕ is compatible with what the speaker knows Δ". This definition resonates the account provided in [4]. The first conjunct (i) expresses the teleological optimality of the prejacent with respect to the internal antecedent. The second conjunct (ii) captures the requirement that action-relevant desire must be realistic: i.e., the goal must be compatible with what is known. Given a context of utterance Δ, a speaker may know of some of the addressee's actual action-relevant desires, we let $\Sigma_\Delta^{D^2} = \{\theta | D^2\theta \in \Delta\}$ denote the set of the addressee's actual goals and call θ a goal whenever $D^2\theta$. In what follows, we slightly abuse notation and write Δ and $\Sigma_\Delta^{D^2}$ for the conjunction of formulae in Δ and $\Sigma_\Delta^{D^2}$, respectively. Let the (teleological) *anankastic conditional* (tac) be defined as:[4]

$$(\Delta, D\phi) \Rightarrow_{tac} O\psi \;:=\; (\Delta, D^2\phi) \Rightarrow O^1(\psi, \Sigma_\Delta^{D^2} \wedge \phi) \;\;\wedge\;\; \Diamond(\Delta \wedge \phi) \tag{4}$$

Applying (4) to premise $\mathsf{P1}$ of the Tobacco shop scenario $(\Delta, D\texttt{smoke}) \Rightarrow_{tac} O\texttt{buy}$ gives us the following formal definition:

$$(\Delta, D^2\texttt{smoke}) \Rightarrow O^1(\texttt{buy}, \Sigma_\Delta^{D^2} \wedge \texttt{smoke}) \;\wedge\; \Diamond(\Delta \wedge \texttt{smoke}) \tag{5}$$

Informally, (5) reads "in the most stereotypical worlds in which Δ and $D^2\texttt{smoke}$ are the case, $\texttt{buy}$ proves teleological optimal given the optimal realisation of the known goals $\Sigma_\Delta^{D^2}$ together with the goal of smoking". Let us look at some logical consequences of definition (4).

Conflicting and non-conflicting goals. Conflicting goals relate to a part of the compositionality problem of anankastics: the addressee's actual action-relevant desires should not matter in the analysis, *unless* these are known to the speaker, see [4]. In La, the issue is accounted for in the same manner as in [4]: only when action-relevant desires are known in Δ, they will be taken into consideration. There are two cases of possible conflict: First, when the desire $D^2\phi$ is incompatible with context Δ. For instance, suppose $\Delta_1 = \{D^2\texttt{health}, \Box(\texttt{smoke} \rightarrow \neg\texttt{health})\}$, then the anankastic

[4] On the left side of (4) we leave D and O underspecified, but the index 'tac' on $\Rightarrow_{tac}$ tells us to interpret the involved modalities as D^2 and O^1 specified on the right side of (4).

$(\Delta_1, D\mathtt{smoke}) \Rightarrow_{tac} O\mathtt{buy}$ is not satisfiable (cf. c_1 p of Def.3.2). Second, when the goal ϕ in $D^2\phi$ is incompatible with Δ. For instance, when you know the shops are closed, and the only chance of smoking would be when the shops are open: given $\Delta_2 = \{\neg\mathtt{open}, \Box(\mathtt{smoke} \rightarrow \mathtt{open})\}$, the second conjunct $\Diamond(\Delta_2 \wedge \mathtt{smoke})$ of the anankastic conditional (4) becomes inconsistent.

Failure of strengthening of the antecedent (SA). In line with [4], there are two ways in which SA may fail: (i) through strengthening that makes the antecedent inconsistent with what is known, and (ii) through strengthening that selects other most stereotypical worlds. As an example, suppose we don't know whether the shops are open today. By later strengthening the antecedent with $\neg\mathtt{open}$, we may obtain a different set of most stereotypical worlds. Failure of SA is guaranteed by the non-monotonic nature of the $\Rightarrow$ modality and its consistency requirement (cf. f_3 of Def.3.2).

The teleological near-anankastic conditional. Near-anankastic conditionals come in different shapes, depending on what readings of the ambiguous 'desire' and 'must' modalities are assigned to the conditional's antecedent and consequent, respectively. Let the *teleological* near-anankastic conditional (tnc) be defined accordingly:

$$(\Delta, D\phi) \Rightarrow_{tnc} O\psi := (\Delta, D^1\phi) \Rightarrow O^1(\psi, \Sigma_{\Delta}^{D^2} \wedge D^1\phi) \tag{6}$$

The formal definition reads: "all the most stereotypical worlds consistent with Δ in which $D^1\phi$ holds are such that the optimal realization of all the addressee's known goals, together with $D^1\phi$, also realize ψ". The presence of $D^1\phi$ in $O^1(\psi, \Sigma_{\Delta}^{D^2} \wedge D^1\phi)$ is important: We take the mere-desire for ψ as a *cause* for the necessitated consequent. In contrast to (4) where optimality is conditioned on the realization of the antecedent's goal, we condition (6) on the desire itself. See [4] for a discussion. The second premise P2 of the Tobacco shop scenario can be assigned this form. Suppose we know that the addressee has an action-relevant desire for $D^2\mathtt{health} \in \Delta$, that smoking is not healthy $\Box(\mathtt{smoke} \rightarrow \neg\mathtt{health})$, and that buying cigarettes together with a mere-desire to smoke will lead to smoking $\Box(\mathtt{buy} \wedge D^1\mathtt{smoke} \rightarrow \mathtt{smoke})$. In that case, the antecedent $D^1\mathtt{smoke}$ together with $\mathtt{buy}$ will lead to a conflict with the optimal realization of the addressee's known desire $D^2\mathtt{health} \in \Delta$. We write,

$$(\Delta, D^1\mathtt{smoke}) \Rightarrow O^1(\neg\mathtt{buy}, \Sigma_{\Delta}^{D^2} \wedge D^1\mathtt{smoke}) \tag{7}$$

For teleological near-anakastic conditionals we likewise have failure of SA. Howevever, note that (6) is not subject to a realism condition due to the presence of a mere desire D^1 in the antecedent.

Deontic near-anankastics with action-relevant desires. We introduce deontic counterparts to the teleological (near-)anankastics. Following [4], deontic near-anankastics emerge when the conditional does not have a purpose reading (e.g., when 'not buying does not serve the purpose of 'smoking'), but a deontic reading of the consequent 'must'. The structure of the first deontic conditional, with an action-relevant reading, is similar to that of the anankastic conditional. The main difference is that in evaluating the hypothesized goal ϕ in $D^2\phi$, we are not concerned with what is deontically optimal given the realization of all the agent's action-relevant desires, but only with what is deontically implied when the goal ϕ is *actualized* given those stereotypical worlds in which Δ and $D^2\phi$ hold (cf. [4]). This is reflected in how the antecedent influences the consequent in (8). To illustrate, think of an agent with a desire to smoke who can either buy or steal cigarettes. Since stealing is forbidden, it must deontically be the case that if she smokes, then she bought the cigarettes. We formalize deontic near-anankastics (dac) with action-relevant desires accordingly:

$$(\Delta, D\phi) \Rightarrow_{dac} O\psi := (\Delta, D^2\phi) \Rightarrow O^2(\psi, \phi) \ \wedge \ \Diamond(\Delta \wedge \phi) \tag{8}$$

Since we are dealing with action-relevant desires, the realism clause is preserved.

Deontic near-anankastics with mere-desires. This conditional is similar to (6). The main difference is again that we are not concerned with what is deontically optimal given the realization of the agent's action-relevant desires, but only with what is deontically implied when the agent has the mere-desire expressed in the antecedent. That is, the occurrence of 'want' is not vacuous but the actual cause of the obligation (cf. [4]). We formalize deontic near-anankastics (dnc) with mere-desires as:

$$(\Delta, D\phi) \Rightarrow_{dnc} O\psi := (\Delta, D^1\phi) \Rightarrow O^2(\psi, D^1\phi) \tag{9}$$

An example of (9) would be when there is an obligation not to smoke. Then, in all deontically optimal worlds where you do not smoke, but desire to smoke, you do not buy cigarettes (since buying, together with a desire to smoke, stereotypically implies smoking). Perhaps less common, deontic conditionals with mere-desires also arise in CTD-scenarios in which desires are forbidden.

For both deontic near-anankastic conditionals (8) and (9) SA fails.

3.3 Ambiguity, (near-)anankastics, and pragmatics

The four readings show that in the consequent, different use is made of the context and the desire modality occurring in the antecedent. This interaction between consequent, antecedent, and context is reflected in the different interpretations of (near-)anankastic conditionals. As observed in Section. 2, there is no difference between the four types of conditionals when we look at "if you want ϕ, you must do ψ". Still, we can differentiate them through linguistic analysis. In particular, the four definitions are differentiated through (i) the role of the context and (ii) the interpretation of the involved hyper-modalities.

Note that we take the antecedent to do double duty: it serves as a "restrictor" of the modal operator, but also conditionalises the modal claim to an assumption. The Tobacco shop scenario illustrates that this is not only desirable, but even necessary. It is normally assumed that an if-clause either restricts an operator, or functions as a supposition. However, this would make P1 and P2 indistinguishable. The double duty of the antecedent is motivated by the fact that only when we consider the consequent as well, we can properly distinguish anankastics from near-anankastics. This is in line with [4].

So far we *assumed pragmatics*: that is, we assumed that we know with which interpretations of 'desire', 'must', and the conditional we are dealing, prior to formalization. Often, we don't have access to a determined interpretation and ambiguity remains. The question is, can we reason with such conditionals even though we don't have a definite interpretation? In the next section, we provide a hyper formalism that enables us to represent and reason with ambiguous conditionals and modalities. By internalizing part of the pragmatics, we may formally reduce possible interpretations through explicit interaction between context, antecedent, and consequent in logic.

4 Hyper modalities: interpreting (near-)anankastics

The two conditionals in the Tobacco shop scenario share the general structure 'if you desire ϕ, you must ψ'. We have argued that 'desire', 'must', and the involved conditional are ambiguous and may receive different readings. Such modalities are called *hyper modalities*. In the previous section, we discussed four possible readings of 'if you desire ϕ, you must ψ'. There, we used distinct modalities for the different readings of 'desire' (D^1 and D^2) and 'must' (O^1 and O^2), and more importantly we *assumed* access to the correct readings of these conditionals and their modalities, prior to their formalization.

We present a way to make ambiguity and interpretation part of the logic, for this we will use the hyper modality framework, as developed in [7]. We introduce the hyper modalities $\mathbb{D}$ and $\mathbb{O}$ to represent the ambiguous 'desire' and 'must'. Such hyper modalities may receive different semantic interpretations depending on their context of evaluation (but under other contexts ambiguity may persist). In Section. 5, we will deploy the formalism to disambiguate and reason with the involved modalities in the Tobacco shop scenario. We point out that the reader may temporarily skip this technical section and first consult the hyper-modal analysis of the Tobacco shop scenario in Section. 5.

4.1 Preliminaries: a brief introduction to hyper-modalities

Why do we need hyper-modalities? Such modalities occur in natural language: for example, "soon p will be true". The reading of 'soon' depends on time: In the 19th century 'soon' could have meant within a week, whereas nowadays 'soon' would mean within 24 hours. Another example, already discussed, is the context dependence of the meaning of 'must' which may refer (among others) to logical necessity, epistemic certainty, and deontic optimality. To represent the linguistic distinctions that may occur in certain contexts, we need to allow for 'must' (and other modalities) to have several semantic interpretations (e.g., S5 for epistemic certainty, but KD for deontic optimality). Hence, in contrast to standard modal logic approaches, we need modalities which do not have a fixed meaning, but *receive their meaning* through evaluation in a context.

4.2 From neighbourhood semantics to hyper modality semantics

Before moving to the multi-modal setting, we introduce the formalism by considering an example language with a single modality $\mathbb{M}$, for 'must'. Let $\mathcal{N}_{\mathbb{M}}$ be a neighbourhood function from worlds $w \in W$ to sets of subsets: $\mathcal{N}_{\mathbb{M}} : W \mapsto \mathcal{P}(W)$. Semantics of atoms and the connectives $\neg$ and $\vee$ are defined as usual, and for $\mathbb{M}$ we adopt $w \models \mathbb{M}\phi$ iff $\|\phi\| \in \mathcal{N}_{\mathbb{M}}(w)$.

We turn $\mathbb{M}$ into a hyper-modality if we allow for each world $w \in W$ an *option* of neighbourhoods functions $\mathcal{N}^1_{\mathbb{M}}(w), \mathcal{N}^2_{\mathbb{M}}(w), ..., \mathcal{N}^n_{\mathbb{M}}(w)$. We call these options the different *modes* of modality $\mathbb{M}$ and denote them by $\Psi^i(w, \mathbb{M}) = \mathcal{N}^i_{\mathbb{M}}(w)$ (for $i \in \{1, ..., n\}$). That is, $\Psi^i(w, \mathbb{M})$ denotes a possible mode for interpreting $\mathbb{M}$ at w. Let $\mathcal{M}odes$ be the set of modes Ψ^i for $i \in \{1, ..., n\}$. Since a modality may have various possible modes, we need a table function, $\underset{\sim}{f} : \mathcal{M}odes \mapsto \mathcal{P}(\mathcal{M}odes)$. So, for each $\Psi^i \in \mathcal{M}odes$, $\underset{\sim}{f}(\Psi^i)$ denotes the set of option modes $\{\Psi^k, ..., \Psi^l\}$ (with $1 \leq l \leq k \leq n$). Last, the semantic clause of a hyper modality $\mathbb{M}$ is relativized to the use of modes Ψ^i, denoted by $\models_{\Psi^i}$. We have, for all $w \in W$:

$$w \models_{\Psi^i} \mathbb{M}\phi \text{ iff for some mode } \Psi^j \in \underset{\sim}{f}(\Psi^i),\ \{v \mid v \models_{\Psi^j} \phi\} \in \mathcal{N}^i_{\mathbb{M}}(w) \tag{10}$$

Hence, $\mathbb{M}\phi$ is satisfiable at w at mode Ψ^i, whenever there is a mode $\Psi^j \in \underset{\sim}{f}(\Psi^i)$ (possibly several) for $\mathbb{M}$ such that $\{v \mid v \models_{\Psi^j} \phi\}$ is in the $\mathbb{M}$-neighborhood for mode Ψ^j. Note that modes are only relevant for evaluating hyper-modalities.

Let us consider an example. We formalize the utterance "it does not rain, but it must rain" as $\neg\texttt{rain} \wedge \mathbb{M}\texttt{rain}$. Let there be two modes for $\mathbb{M}$: for any w, let $\Psi^{deo}(w, \mathbb{M}) = \mathcal{N}^{deo}_{\mathbb{M}}(w)$ s.t. $\mathcal{N}^{deo}_{\mathbb{M}}(w)$ does not contain $\emptyset$ (i.e., Ψ^{deo} interprets $\mathbb{M}$ deontically by excluding inconsistencies) Let $\Psi^{epi}(w, \mathbb{M}) = \mathcal{N}^{epi}_{\mathbb{M}}(w)$ s.t. $\mathcal{N}^{epi}_{\mathbb{M}}(w)$ is restricted to all the sets containing the world w (i.e., Ψ^{epi} takes $\mathbb{M}$ as some sort of epistemic certainty: 'if ϕ is epistemically certain, ϕ must be true now'). Let $\underset{\sim}{f}(\Psi^{deo}) = \underset{\sim}{f}(\Psi^{epi}) = \{\Psi^{deo}, \Psi^{epi}\}$, which means that in both the deontic and epistemic mode, $\mathbb{M}\phi$ may be interpreted deontically, as well as epistemically. How do we evaluate $\neg\texttt{rain} \wedge \mathbb{M}\texttt{rain}$ at w? We need to pick a starting mode. Suppose it is Ψ^{deo}. Hence, $w \models_{\Psi^{deo}} \neg\texttt{rain} \wedge \mathbb{M}\texttt{rain}$ iff $w \not\models_{\Psi^{deo}} \texttt{rain}$ and $w \models_{\Psi^{deo}} \mathbb{M}\texttt{rain}$. The last conjunct in equivalent to $\{v \in W \mid v \models_{\Psi^i} \mathbb{M}\texttt{rain}\} \in \mathcal{N}^i_{\mathbb{M}}(w)$ for some $\Psi^i \in \{\Psi^{epi}, \Psi^{deo}\}$. If $\Psi^i = \Psi^{epi}$, then $w \in \{v \in W \mid v \models_{\Psi^{epi}} \texttt{rain}\}$, but $w \in \{v \in W \mid v \models_{\Psi^{deo}} \neg\texttt{rain}\}$ (modes do not apply to atoms). We have a contradiction. Hence, the ambiguous $\mathbb{M}\texttt{rain}$ cannot be interpreted epistemically given $\neg\texttt{rain}$ (whether it can be interpreted deontically, remains to be determined). See [7] for further examples.

4.3 Interpreting (near-)anankastics using hyper-modalities

Since the conditional 'if you want ϕ, you must ψ' depends on the ambiguous 'want' and 'must', the conditional $\Rightarrow$ is likewise ambiguous. The hyper modalities that we will consider are thus 'desire' $\mathbb{D}$, 'must' $\mathbb{O}$ and 'conditional' $\Rightarrow$. We build our hyper modality setting on top of the La-neighbourhood semantics of Def.3.2. The hybrid language $\mathcal{L}_{\mathsf{LaH}}$ is defined through the following BNF:

$$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid D^1\phi \mid D^2\phi \mid O^1(\phi, \phi) \mid O^2(\phi, \phi) \mid \Box\phi \mid (\phi, \phi) \Rightarrow \phi \mid \mathbb{D}\phi \mid \mathbb{O}\phi$$

with $p \in \mathsf{Atoms}$. The language La properly extends $\mathcal{L}_{\mathsf{LaH}}$ for the reason that, in evaluating a (near-)anankastic, we involve a context Δ which may contain information about unambiguous desires and obligations. Note that $\mathbb{O}\phi$ is monadic but will be interpreted dyadically in the hyper-setting, conditioning it on a given context (if there is no context we evaluate $\mathbb{O}\phi$ conditional on $\top$).

To facilitate readability, we write $\|\phi\|_{\Psi^i} = \{v \in W \mid v \models_{\Psi^i} \phi\}$ to indicate the presence of a mode Ψ^i for evaluating ϕ. Furthermore, we explicitly index the modes Ψ^i with the formula's arguments. As an example, $w \models_{\Psi^i} \mathbb{O}(\phi, \psi)$ means $\mathbb{O}(\phi, \psi)$ is evaluated with respect to the mode $\Psi^i_{\phi,\psi}(w, \mathbb{O})$. Most formulae will be evaluated with respect to what we call the *common* mode, denoted by Ψ^{co}.

We are interested in mode shifts that occur when evaluating conditionals of the form $(\Delta, \mathbb{D}\phi) \Rightarrow \mathbb{O}\psi$. The four interpretations of (near-)anankastic conditionals (Section. 3) are in fact *modes* for interpreting "if you want ϕ, you must ψ": i.e., anankastics Ψ^{tac}, near-anankastics Ψ^{tnc}, deontic anankastics Ψ^{dac}, and deontic near-anankastics Ψ^{dnc}. For instance, when evaluating in *anankastic* mode Ψ^{tac}, $\mathbb{D}\phi$ is interpreted as an action-relevant D^2, and $\mathbb{O}\psi$ via a teleological optimization O^2. Let us make these modes formally precise:

Definition 4.1 A *hyper* La-frame is a tuple $F = \langle W, \mathcal{N}_{D^1}, \mathcal{N}_{D^2}, \mathcal{N}_{O^1}, \mathcal{N}_{O^2} \mathcal{N}_{\Rightarrow}, \mathcal{M}odes \rangle$, where $W \neq \emptyset$ is a non-empty set of worlds $w, v, u, ..$ and $\mathcal{N}_i$ ($i \in \{D^2, D^1, O^1, O^2, \Rightarrow\}$) are neighbourhood functions as defined in Def.3.2. F satisfies the constraints from Def.3.2. Let the set of modes be $\mathcal{M}odes = \{\Psi^{co}, \Psi^{tac}, \Psi^{tnc}, \Psi^{dac}, \Psi^{dnc}\}$ (defined in Def.4.2, Def.4.4, Def.4.3, resp). The assignment $\underset{\sim}{f}^i$ of modes ($i \in \{1, 2, 3\}$) is defined as:

- $\underset{\sim}{f}^1 : \mathcal{M}odes \mapsto \mathcal{P}(\mathcal{M}odes)$ (for monadic $\mathbb{D}$)
- $\underset{\sim}{f}^2 : \mathcal{M}odes \mapsto \mathcal{P}(\mathcal{M}odes \times \mathcal{M}odes)$ (for dyadic $\mathbb{O}$)
- $\underset{\sim}{f}^3 : \mathcal{M}odes \mapsto \mathcal{P}(\mathcal{M}odes \times \mathcal{M}odes \times \mathcal{M}odes)$ (for triadic $\Rightarrow$)

A *hyper* La-model M consists of a *hyper*-frame F with a valuation V.

The function $\underset{\sim}{f}^i$ in Def.4.1 determines, at a given mode, the possible modes available for evaluating a given modal formula. By default, we take as the *starting mode* for evaluating formulae the mode Ψ^{co}.

Definition 4.2 Given $\underset{\sim}{f}^1$ of Def.4.1, we specify the following modes for $\mathbb{D}$:

(i) for $i \in \{tac, dac\}$, $\Psi^i_\phi(w, \mathbb{D})$ is $\|\phi\|_{\Psi^j} \in \mathcal{N}_{D^2}(w)$ with $\underset{\sim}{f}^1(\Psi^i) = \Psi^j$

(ii) for $i \in \{tnc, dnc\}$, $\Psi^i_\phi(w, \mathbb{D})$ is $\|\phi\|_{\Psi^j} \in \mathcal{N}_{D^1}(w)$ with $\underset{\sim}{f}^1(\Psi^i) = \Psi^j$

(iii) for $i = co$, $\Psi^i_\phi(w, \mathbb{D})$ is $\|\phi\|_{\Psi^j} \in \mathcal{N}_{D^2}(w)$ or $\|\phi\|_{\Psi^j} \in \mathcal{N}_{D^1}(w)$ with $\underset{\sim}{f}^1(\Psi^i) = \Psi^j$

With $\underset{\sim}{f}^1(\Psi^i) = \{\Psi^{co}\}$ for each $i \in \{co, tac, tnc, dac, dnc\}$.

To illustrate, consider (i) of Def.4.2: when evaluating in the teleological anankastic mode Ψ^{tac}, we interpret 'desire' $\mathbb{D}\phi$ as an action-relevant desire D^2 and interpret the internal goal ϕ in common mode. Condition (iii) states that, in the common mode, an ambiguous $\mathbb{D}\phi$ is satisfiable whenever it is satisfiable as a mere-desire or an action-relevant desire (possibly both).

For conditionals of the form $(\Delta, \mathbb{D}\phi) \Rightarrow \mathbb{O}\psi$ we find four possible (near-)anankastic interpretations of $\Rightarrow$. When a conditional is of the form $(\Delta, \theta) \Rightarrow \chi$ such that $\theta \neq \mathbb{D}\phi$ or $\chi \neq \mathbb{O}\psi$, we evaluate $\Rightarrow$ as a regular conditional.

Definition 4.3 Given $\underset{\sim}{f}^3$ of Def.4.1, we specify the following modes for $\Rightarrow$.

(i) If $\phi = \mathbb{D}\phi'$ and $\psi = \mathbb{O}\psi'$, then $\Psi^{co}_{\Delta,\phi,\psi}(w, \Rightarrow)$ is $(\|\Delta\|_{\Psi^j}, \|\phi\|_{\Psi^k}, \|\psi\|_{\Psi^l}) \in \mathcal{N}_{\Rightarrow}(w)$, for some $(\Psi^j, \Psi^k, \Psi^l) \in \underset{\sim}{f}^3(\Psi^{co}) \setminus \{(\Psi^{co}, \Psi^{co}, \Psi^{co})\}$

(ii) If $\phi \neq \mathbb{D}\theta$ or $\psi \neq \mathbb{O}\chi$, then $\Psi^{co}_{\Delta,\phi,\psi}(w, \Rightarrow)$ is $(\|\Delta\|_{\Psi^{co}}, \|\phi\|_{\Psi^{co}}, \|\psi\|_{\Psi^{co}}) \in \mathcal{N}_{\Rightarrow}(w)$, for $(\Psi^{co}, \Psi^{co}, \Psi^{co}) \in \underset{\sim}{f}^3(\Psi^{co})$

With $\underset{\sim}{f}^3(\Psi^i) = \{(\Psi^{co}, \Psi^{co}, \Psi^{co}), (\Psi^{co}, \Psi^{tac}, \Psi^{tac}), (\Psi^{co}, \Psi^{tnc}, \Psi^{tnc}), (\Psi^{co}, \Psi^{dac}, \Psi^{dac}), (\Psi^{co}, \Psi^{dnc}, \Psi^{dnc})\}$ for $i \in \{co, tac, tnc, dac, dnc\}$.

Def.4.3 ensures that conditionals are only evaluated in Ψ^{co} mode, namely, (near-)anankastic modes are reserved for the hyper modalities $\mathbb{D}$ and $\mathbb{O}$ occurring *within* such a conditional. Only $\mathbb{D}$ and $\mathbb{O}$ can be evaluated in (near-)anankastic modes, which are modes that arise by identifying a hyper-conditional of the form $(\Delta, \mathbb{D}\phi) \Rightarrow \mathbb{O}\psi$. Hence, when evaluating $\mathbb{O}$ in a (near-)anankastic mode, we *come from* a mode that interprets a conditional: consequently, we have additional information (an antecedent and a context) at our disposal that facilitates interpreting $\mathbb{O}$. This is reflected in Def.4.4.

Definition 4.4 Given $\underset{\sim}{f}^2$ of Def.4.1, we specify the following modes for $\mathbb{O}$.

(i) $\Psi^{tac}_{\Delta,\mathbb{D}\phi,\mathbb{O}\psi}(w, \mathbb{O})$ is $(\|\psi\|_{\Psi^j}, \|\Sigma^{D^2}_{\Delta} \wedge \phi\|_{\Psi^k}) \in \mathcal{N}_{O^1}(w)$ with $(\Psi^j, \Psi^k) \in \underset{\sim}{f}(\Psi^{tac})$

(ii) $\Psi^{tnc}_{\Delta,\mathbb{D}\phi,\mathbb{O}\psi}(w, \mathbb{O})$ is $(\|\psi\|_{\Psi^j}, \|\Sigma^{D^2}_{\Delta}\|_{\Psi^k} \cap \|\mathbb{D}\phi\|_{\Psi^{tnc}}) \in \mathcal{N}_{O^1}(w)$ with $(\Psi^j, \Psi^k) \in \underset{\sim}{f}(\Psi^{tnc})$

(iii) $\Psi^{dac}_{\Delta,\mathbb{D}\phi,\mathbb{O}\psi}(w, \mathbb{O})$ is $(\|\psi\|_{\Psi^j}, \|\phi\|_{\Psi^k}) \in \mathcal{N}_{O^2}(w)$ with $(\Psi^j, \Psi^k) \in \underset{\sim}{f}(\Psi^{dac})$

(iv) $\Psi^{dnc}_{\Delta,\mathbb{D}\phi,\mathbb{O}\psi}(w, \mathbb{O})$ is $(\|\psi\|_{\Psi^j}, \|\mathbb{D}\phi\|_{\Psi^k}) \in \mathcal{N}_{O^2}(w)$ with $(\Psi^j, \Psi^k) \in \underset{\sim}{f}(\Psi^{dnc})$

(v) $\Psi^{co}_{\top,\mathbb{O}\psi}(w, \mathbb{O})$ is $(\|\psi\|_{\Psi^j}, \|\top\|_{\Psi^k}) \in \mathcal{N}_{O^1}(w)$ or $(\|\psi\|_{\Psi^j}, \|\top\|_{\Psi^k},) \in \mathcal{N}_{O^2}(w)$ with $(\Psi^j, \Psi^k) \in \underset{\sim}{f}(\Psi^{co})$

And $\underset{\sim}{f}^2(\Psi^i) = \{(\Psi^{co}, \Psi^{co}\}$ for $i \in \{co, tac, tnc, dac\}$ and $\underset{\sim}{f}^2(\Psi^{dnc}) = \{(\Psi^{dnc}, \Psi^{co})\}$. (Note that for (tnc) and (dnc), we require that $\mathbb{D}\phi$ in the second argument of $\mathbb{O}$ is interpreted as D^1.)

Consider (i) in Def.4.4, when evaluating $\mathbb{O}$ in anankastic mode Ψ^{tac}, we check whether in those cases where the agent's known action-relevant desires $D^2\theta \in \Delta$ have been optimally realized, together with the realization of ϕ, we find that ψ is the case. Hence, in anankastic mode Ψ^{tac}, we treat the antecedent $\mathbb{D}\phi$ as if the agent has an action-relevant desire $D^2\phi$, and evaluate $\mathbb{O}\psi$ teleologically as $O^1\psi$, while conditioning it explicitly on the context Δ.

Last, we define the semantics of hyper modalities $\mathbb{D}$, $\mathbb{O}$, and $\Rightarrow$. Note that modes are *only activated* whenever we encounter a hyper modality in a formula.

Definition 4.5 Let M be a hyper La-model of Def.4.1. For every $w \in W$ we have the regular clauses for non-hyper modalities of Def.3.3, extended with:

- $w \models_{\Psi^i} \mathbb{D}\phi$ iff $\mathbb{D}\phi$ is satisfied at w for some $\Psi^j \in \underset{\sim}{f}^1(\Psi^i)$ of Def.4.2.
- $w \models_{\Psi^i} \mathbb{O}(\phi, \psi)$ iff $\mathbb{O}(\phi, \psi)$ is satisf. at w for some $\Psi^j \in \underset{\sim}{f}^2(\Psi^i)$ of Def.4.4.
- $w \models_{\Psi^i} (\theta, \phi) \Rightarrow \psi$ iff $(\theta, \phi) \Rightarrow \psi$ is satisf. at w for some $\Psi^j \in \underset{\sim}{f}^3(\Psi^i)$ of Def.4.3.

A formula is La-*satisfiable* if there is an La-model M with $w \in W$ of M and there is a mode $\Psi^i \in \mathcal{M}odes$ s.t. $M, w \models_{\Psi^i} \phi$.

For any ambiguous conditional $\theta = (\Delta, \mathbb{D}\phi) \Rightarrow \mathbb{O}\psi$ the hyper setting gives us the following: If *only* Ψ^{ac} is satisfiable we say θ is an *anankastic conditional.* If *only* Ψ^{nc} is satisfiable θ is a *teleological near-anankastic*. If *only* Ψ^{da} is satisfiable, we say θ is a action-relevant *deontic near-anankastic conditional.* If *only* Ψ^{dn} is satisfied θ is called a mere-desire *deontic near-anankastic conditional.* If several of (i)-(iv) are satisfied, the resulting interpretation is a disjunction reflecting the possible readings of $(\Delta, \mathbb{D}\phi) \Rightarrow \mathbb{O}\psi$ given Δ. If neither is satisfiable, we say the $(\Delta, \mathbb{D}\phi) \Rightarrow \mathbb{O}\psi$ has no interpretation given Δ, and hence is false. In interpreting hyper formulae of the form $(\Delta, \mathbb{D}\phi) \Rightarrow \mathbb{O}\psi$ in modes $\Psi^{ac}, \Psi^{nc}, \Psi^{da}$, and Ψ^{dn}, we employ the same semantic interpretations as used for the four formally defined conditionals (4), (6), (8), (9), of Section. 3.

The main difference between Section. 3 and the hyper-approach presented here, is that (a) we internalise the interpretation procedure (i.e., part of the pragmatics) through using hyper modalities and corresponding modes, and (b) we leave open the possibility that a conditional remains ambiguous (i.e., several modes may be satisfiable given a context Δ). As a consequence of (a) and (b), we can logically reason with ambiguous conditionals, such as P1 and P2 of the Tobacco shop scenario, without assuming a definite linguistic interpretation. We can use logic to determine, given a certain context, which interpretations of ambiguous (near-)anankastic conditionals are excluded, and which are jointly satisfiable. Let us look at the Tobacco shop scenario again.

5 Disambiguation and the Tobacco shop scenario

For sentences such as "if you want ϕ, you must ψ" the hyper-setting can help reducing ambiguity by determining which interpretations (i.e., modes) are excluded given certain contexts. To illustrate this, we have another look at the Tobacco shop scenario and consider two possible contexts. First, we recall the remarks by Prof. Pragmatics and Prof. Restraint, respectively:

If S wants to smoke, then S must buy cigarettes. (P1)

If S wants to smoke, then S must not buy cigarettes. (P2)

Using the hyper-modalities for 'desire' $\mathbb{D}$, 'must' $\mathbb{O}$, and the conditional '$\Rightarrow$', we obtain the following hyper-modal readings (adding $\top$ for an empty context):

$$(\top, \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\texttt{buy} \tag{11}$$

$$(\top, \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\neg\texttt{buy} \tag{12}$$

Suppose that at this point we do not yet know which readings, or contexts, Pragmatics and Restraint assign to their utterances. Can we already derive something from the joint utterance of (11) and (12)? The answer is yes. The hyper-modal setting tells us that (11) and (12) cannot be jointly satisfied under the same mode. For instance, if we interpret both formulae as anankastic conditionals (*tac*) the conjunction is not satisfiable (models are assumed to be hyper models from Def.4.1):

$$\text{For any } w \in W,\ w \not\models_{\Psi^{co}} (\top, \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\neg\texttt{buy} \text{ or } w \not\models_{\Psi^{co}} (\top, \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\neg\texttt{buy} \text{ for } \Psi^{tac} \in \underset{\sim}{f}(\Psi^{co}) \tag{13}$$

In short, (13) depends on the consistency requirement on 'must' O^1 under consistent conditions, together with the exclusion of impossible conditionals (d_2, f_1, and f_2 of Def.4.1). Similar reasoning excludes identical interpretations of (11) and (12) for any of the four modes Ψ^i, $i \in \{tac, tns, dac, dnc\}$. (For space reasons, all semantic proofs will be omitted.) Hence, the formalism allows us to conclude that P1 and P2 must have *distinct* (near-)anankastic readings if they are to be jointly satisfiable.

Recall that P1 is commonly taken as an anankastic conditional, that is, 'buying' proves teleologically optimal for realizing the goal of 'smoking'. If we take (14) as given, what additional conclusions can we draw concerning P2?

$$(\top, D^2\texttt{smoke}) \Rightarrow O^1\texttt{buy} \tag{14}$$

If Pragmatics and Restraint agree on the fact that Smoke has an action-relevant desire to smoke, the only mode in which (12) may be satisfied is Ψ^{dac}. So far, we were able to draw some minimal conclusions about P1 and P2 without assuming any additional context, that is, these conclusions were drawn from the logical behaviour for the different modes of hyper-modalities $\mathbb{D}$ and $\mathbb{O}$.

Now, suppose Prof. Pragmatics asks Restraint to to explain herself. The latter recalls that Dr. Smoke has an action-relevant desire to stay healthy pointing out that smoking will obstruct that goal. We obtain the context $\Delta = \{D^2\texttt{health}, \Box(\texttt{smoke} \to \neg\texttt{health})\}$ and update the formalisation of P2:

$$(\Delta, \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\neg\texttt{buy} \tag{15}$$

Independent of how we interpret P1, the additional context for P2 excludes the interpretation that (15) is an anankastic conditional: The action-relevant desire to be healthy $D^2\texttt{health}$ (with $\texttt{health} \in \Sigma_{\Delta}^{D^2}$), cannot be realised together with an action-relevant interpretation of $\mathbb{D}\texttt{smoke}$, namely, $D^2\texttt{smoke}$. In brief, goals expressed by an agent's action-relevant desires should be jointly realisable (cf. f_1, f_2 of Def. 4.1). The result is expressed in (16).

$$\text{For any } w{\in}W,\ w \not\models_{\Psi^{co}} (\Delta, \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\neg\texttt{buy}, \text{ for } \Psi^{tac} \in \underset{\sim}{f}(\Psi^{co}) \tag{16}$$

The above does not imply that an action-relevant reading of $\mathbb{D}\texttt{smoke}$ is impossible for (12): a *deontic* reading of the consequent $\mathbb{O}\neg\texttt{buy}$ interacts differently with the context and thus allows for other desire statement in the antecedent (cf. the discussion of (8) in Section. 3). Furthermore, we find that given Δ the realism requirement imposed on action-relevant (near-)anankastics in general is still satisfiable, i.e., if you smoke, you can still have an action-relevant desire to be healthy, but the latter goal cannot be attained.

Suppose another context Δ' in which Prof. Restraint recalls Dr. Smoke's promise to buy some cigarettes for a friend. Moreover, suppose she points out that keeping the promise $\texttt{prom}$ is Smoke's duty, *irrespective* of whether he desires to smoke $D^1\texttt{smoke}$ or actually does so, $\texttt{smoke}$. In other words, if Smoke keeps his promise, he will buy cigarettes $\Box(\texttt{prom} \rightarrow \texttt{buy})$. We obtain the new context $\Delta' = \{O^2(\texttt{prom}, \texttt{smoke}), O^2(\texttt{prom}, D^1\texttt{smoke}), \Box(\texttt{prom} \rightarrow \texttt{buy})\}$.

$$\text{For any } w \in W,\ w \not\models_{\Psi^{co}} (\Delta', \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\neg\texttt{buy}, \text{ for } \Psi^{dac}, \Psi^{dnc} \in \underset{\sim}{f}(\Psi^{co}) \tag{17}$$

We find that the conditional expressed in (17) excludes any reading of P2 as a *deontic* near-anankastic conditional, i.e., under either desire reading. Namely, given Δ', the obligation to keep one's promise will conflict with the readings $O^2(\neg\texttt{buy}, \texttt{smoke})$ and $O^2(\neg\texttt{buy}, D^1\texttt{smoke})$ since not buying implies breaking one's promise (cf. f_1, f_2, and d_2 of Def. 4.1). Given Δ' and the anankastic reading of P1 (14), the only reading of $(\Delta', \mathbb{D}\texttt{smoke}) \Rightarrow \mathbb{O}\neg\texttt{buy}$ which is not necessarily excluded is the teleological near-anankastic reading.

The analysis shows that, through interaction between contexts (such as Δ and Δ') and different interpretations of the antecedent and consequent (D^1 and D^2, respectively, O^1 and O^2) we may formally exclude certain interpretations of ambiguous linguistic expressions such as P1 and P2 of the Tobacco shop scenario. The example shows that certain restrictions on different interpretations of (near-)anankastics serve to reduce ambiguity, e.g., consistency of goals for teleological optimality in (16).

The hyper-modal setting enables us to represent ambiguity, and use formal machinery to (partially) resolve it, thus internalising some of the pragmatics of linguistic interpretations. Some of the benefits of this approach are that (i) we do not need to assume prior to formalisation that all ambiguity is resolved, (ii) we can formalise ambiguous sentences that will receive their interpretation at a later stage, and (iii) we can study those criteria that function as identifiers for rightly interpreting hyper-modalities. Still, future work should be devoted to identifying other conditions that enable us to draw conclusions from hyper-modal formulae concerning 'must', 'desire', and (near)-anankastic conditionals. Another point left unaddressed here is whether the logic of (near-)anankastic conditionals allows for (certain forms of) detachment (cf. [10]).

6 Conclusion and future work

In this work, we related semantics of deontic modality and deontic logic. We discussed the Tobacco shop scenario, highlighting the interaction between consequent, antecedent, and context in interpreting (near-)anankastics. We presented a logic inspired by [4], capturing four (near-)anankastic conditionals, while assuming linguistic interpretation of concrete conditionals prior to formalisation. We extended the formalism to the hyper setting, where hyper-modalities bring ambiguity within the reach of logical analysis: i.e., internalising parts of the interpretation process of modalities, such as 'must' and 'desire'.

Perhaps the most unusual aspect of our approach is that we treat 'context' as part of the syntax of a formula. This means that one and the same natural language conditional must be translated differently in different contexts. This is unusual, since most approaches aim for a systematic analysis that accounts for the way in which the content of the sentence depends on context. Moreover, in Section. 5 we did not fully exploit the additional expressive power that comes with having the context in the language. We plan to do this in future research. For example, under suitable conditions, instead of assuming that we have a prohibition to smoke, we would be able to derive it.

This paper touched on several other points requiring future work: (i) Further the analysis of the Tobacco shop scenario, e.g., by relating it to existing approaches handling contrary-to-duty reasoning, nonmonotonic reasoning, and dynamic logic. (ii) Extend the analysis of hyper-modality (e.g., in the context of NLP). (iii) Investigate other aspects of pragmatics that can be studied in logic (e.g., detachment using nonmonotonic logic).

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