Duty and Sacrifice
A Logical Analysis of the Mīmāṃsā Theory of Vedic Injunctions

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Abstract The Mīmāṃsā school of Indian philosophy has for its main purpose the interpretation of injunctions that are found in a set of sacred texts, the Vedas. In their works, Mīmāṃsā authors provide some of the most detailed and systematic examinations available anywhere of statements with a deontic force; however, their considerations have generally not been registered outside of Indological scholarship. In the present article we analyze the Mīmāṃsā theory of Vedic injunctions from a logical and philosophical point of view. The theory at issue can be regarded as a system of reasoning based on certain fundamental principles, such as the distinction between strong and weak duties, and on a taxonomy of ritual actions. We start by reconstructing the conceptual framework of the theory and then move to a formalization of its core aspects. Our contribution represents a new perspective to study Mīmāṃsā and outlines its relevance, in general, for deontic reasoning.

Keywords Deontic logic · Deontic reasoning · Indian philosophy · Mīmāṃsā

1 Introduction

1.1 Presentation of the topic

The Mīmāṃsā school of Indian philosophy provides an analysis of injunctions (vidhi in Sanskrit) that are found in a set of sacred texts called the Vedas. We use ‘injunction’ in the sense of a statement which enjoins someone to do something, thus expressing a duty. The injunctions with which Mīmāṃsā is concerned are generally injunctions to perform sacrifices (karma, or ‘action’, being used as the technical term for these ritual actions). When we speak of the ‘analysis’ of injunctions, we mean the elaboration of a set of general principles according to which a given injunction might be understood and hence put into action.¹

The foundations of Mīmāṃsā as a philosophical school were laid perhaps in the second century before the common era. Mīmāṃsā provides some of the most detailed and systematic analyses available anywhere of statements with a deontic force. These discussions, however, have generally not been registered outside of Indological scholarship. One way to evaluate their philosophical significance is through a rigorous analysis of the deontic concepts that they employ. Such an analysis can provide important insights about their precise characterization, including the similarities to, and differences from, the way in which

¹ For a short introduction to Mīmāṃsā in general, see Freschi 2017. For injunctions and ritual duties as conceived in Mīmāṃsā, see sections 3–6 of Freschi 2012 and Ollett 2013.
concepts such as ‘obligation’ and ‘permission’ have been defined in other philosophical systems. As a part of our reconstruction of the arguments employed by Mīmāṃsakas (philosophers operating within the Mīmāṃsā school), we introduce a formal system of reasoning focused on fundamental notions involved in these arguments, such as ‘sacrifice’, ‘primary action’, ‘subsidiary action’ and ‘eligibility’. This formal perspective can, in turn, offer new interpretive and philosophical insights into the texts of Mīmāṃsā in which the arguments at issue were articulated.

The formalization of deontic arguments in Mīmāṃsā should be seen as part of an ongoing attempt to use formal approaches to represent arguments and principles of reasoning in Indian philosophical thought. This enterprise has been going on since the beginnings of the study of the history of Indian logic in Europe (see, for instance, Stcherbatsky 1930–1932, Ingalls 1951, Bochenski 1956 and Staal 1967) and the trend has continued up to recent years, especially in regard to Buddhist philosophy of the Madhyamaka school (see, for instance, Garfield and Priest 2003 and Guhe 2017). Another area of application of symbolic tools has been the epistemology and logic of the Dignāga-Dharmakīrti school (see for instance Patil 2009 and Tillemans 2013) and of the Nyāya and Nāvya Nyāya school (see for instance Matilal 1985 and Ganeri 2001). However, no work has systematically investigated the analysis of logical principles and, especially, principles of deontic reasoning in the Mīmāṃsā school. In fact, the natural place to look for the study of deontic concepts in India is Mīmāṃsā, which is however far less studied than other philosophical traditions. Moreover, the few scholars working on Mīmāṃsā by using logical formalizations focused on epistemology and other topics that did not involve deontic concepts (see, for instance, Yoshimizu 2007).

Given the enormous breadth of Mīmāṃsā, a tradition which spans over two thousand years, and includes many thinkers who disagreed with each other, the research agenda of studying the principles of deontic reasoning used by Mīmāṃsākās must proceed on a problem-by-problem basis and must begin with a focus on particular texts. This paper, while addressing one specific, though central, aspect of the Mīmāṃsā theory, will also lay some of the groundwork for further research, and should be taken as a starting point in the logico-philosophical reconstruction of this theory. Furthermore, the present work will enable scholars to contribute the contribution of other important schools of Indian philosophy, such as the Nyāya school and the Dignāga-Dharmakīrti school. Finally, our analysis will deal with issues that are relevant to deontic reasoning in general, such as the difference between strong and weak duties.

1.2 Working plan for the present article

The main part of this article will focus on what we call Common Mīmāṃsā. The label refers to the early history of the philosophical system of Mīmāṃsā, as depicted in its root texts, Jaimini’s Mīmāṃsāsūtra ‘Maxims on Vedic exegesis’ or ‘Exegetic Aphorisms’ (possibly 2nd c. BCE) and in Śabara’s Bhāṣya ‘Commentary’ (possibly 3rd c. CE). The specific problem that we address pertains to the classification of sacrifices into fixed, occasional, and elective, that is accepted in Common Mīmāṃsā. On the one hand, the injunctions to perform sacrifices of these three types have an identical linguistic form and they are all regarded as originating a kind of duty. On the other hand, we will see that fixed and occasional sacrifices are compulsory in a way that elective sacrifices are not, and furthermore, the subsidiary actions that form part of an elective sacrifice, and those that form part of a fixed or occasional sacrifice, are also not compulsory in the same way. The taxonomy of ritual actions therefore poses a significant philosophical challenge: how exactly should the various notions of ‘duty’ at play in this taxonomy be characterized, and what accounts for their differences?

As we said, part of our reconstruction consists in employing a formal language. The rendering of complex arguments from the history of philosophy—and in particular from Sanskrit texts composed more than fifteen centuries ago—into a formal language is a difficult enterprise. There is a risk that what is rendered in the formalism will bear little relation to the textual argumentation, either because it is too open-ended and general, or because it is too literalistic and particularistic. However, for a recent and more generic discussion of deontic and related concepts in ancient India, see Majumdar 2017.
abstract, or because important distinctions are omitted. To minimize this risk, we will proceed in two stages. First (section 2), we will offer a descriptive analysis of the phenomenon, as far as we understand it from a set of texts that we consider to represent Common Mīmāṃsā. This will involve setting out the three main categories of sacrificial actions, and introducing some of the general principles that are invoked in differentiating them and eliciting their deontic consequences. Second (section 3), we introduce a formal language based on this taxonomy and build a logical system in which it is possible to represent generic patterns of reasoning related to the Mīmāṃsā theory. We provide a semantic characterization of our system and illustrate some possible variations of it, such as extensions including further logical principles or different logical operators. Finally, we illustrate how the formal systems introduced here are related to a formal system that was previously used to model a debate within Mīmāṃsā (Appendix).

In addition to this, in section 4, we discuss a thesis formulated by an eighth-century Mīmāṃsā author, Maṇḍana Miśra, who attempted to reinterpret the theory of sacrifices found in Common Mīmāṃsā. Maṇḍana thought that all Vedic injunctions can be redescribed in terms of a notion of instrumentality towards a given end, without losing the distinctions that arise from the traditional taxonomy. Maṇḍana’s intervention is ‘reductionist’, both in the sense that it brings the types of duties pertaining to various kinds of sacrificial action under a uniform description, and also in the sense that it reduces deontic concepts to non-deontic concepts. We discuss how these simplifications affect the formalization we provided, and what philosophical consequences they have.

1.3 Mīmāṃsā’s particularities

In this paper we will discuss the deontic concepts employed by authors in the Mīmāṃsā tradition. Here we must draw attention to three features of this tradition, so that the reader may have a clearer idea of what these concepts are, what it means for us to attribute them to Common Mīmāṃsā, and the role that individual authors played in defining and redefining this system. These features distinguish Mīmāṃsā from other traditions of reflection on deontic concepts, and hence should condition our expectations of Mīmāṃsā in comparison to other such traditions, such as Talmudic or Quranic interpretation, Euro-American traditions of moral philosophy, or Confucian ethics.

Deontic concepts are very often invoked in connection with an idea of what is ‘good’ or ‘right’, that is, in connection with ethical concepts. This connection is so weak in Mīmāṃsā that we may think of its deontology and ethics being completely independent from each other: by and large, Mīmāṃsā is concerned with analyzing rituals, the performance of which is enjoined in the Vedas; it is taken for granted that the performance of these rituals is conducive to some notion of ‘the good’, which is, however, not determined in any detail. Moreover, the very fact that an action is enjoined in the Vedas is taken to mean that it cannot be derived from any understanding of ‘the good’ that is independent of the Vedas. One example is the Vedic injunction ‘do not tell a lie’: in Common Mīmāṃsā, this is not a general ethical recommendation, but a specific requirement of someone who is performing the full- and new-moon sacrifices, having exactly the same deontic and ethical status as injunctions, such as ‘pour the ghee into the fire’, which transparently apply only to ritual actions. The duties that we will be examining in this paper, therefore, pertain specifically to the sphere of ritual.

The stated goal of Mīmāṃsā is to interpret the statements of the Vedas, and thus to provide specific guidance for performing the rituals they enjoin. But there was widespread agreement, from a relatively early period, about what the Vedas tell us to do. By contrast, Mīmāṃsā authors often disagreed about the principles by which we are led to these interpretations. Thus, if we only look at the conclusions—that a particular person, in a particular circumstance, has a duty to perform a certain ritual action—it will appear as if Mīmāṃsā authors all agree with each other. But if we look at the reasoning leading up to those conclusions, we see significant differences, since some authors, like Prabhākara (6th-7th c. CE), interpret the Vedas as a set of rules to be followed, while other authors, like Maṇḍana, interpret the Vedas as a set of recipes to reach desired goals.

Finally, Mīmāṃsā, like other traditional knowledge systems in India, operates according to the conceit that all of the doctrines of the system are contained in nuce in the foundational work of the system, namely, Jaimini’s Maxims, and that every subsequent author has merely explained, clarified, or summarized these doctrines. This is hardly more than a conceit, and in fact many authors have introduced radical changes into the system, but very often they have done so by claiming that their interventions...
are merely elaborations of ideas that can be found in earlier works. Accordingly, determining the specific contributions of each author is sometimes difficult, and even authors who have radically revised the system, like Maṇḍana, go out of their way to remain faithful to the distinctions and classifications introduced by earlier authors.

2 Vedic duties in Common Mīmāṃsā

Mīmāṃsā is principally concerned with duties: how they are to be understood and followed, on a practical level, and how they are to be theoretically grounded. But the set of real-world duties is much larger than the set of duties that Mīmāṃsā addresses. Mīmāṃsā is concerned specifically with those duties that are presented in the Vedas, and even more specifically, with those duties presented in the Vedas to perform ritual actions. Hence Mīmāṃsā, literally ‘analysis’, is often called karmamīmāṃsā, ‘the analysis of ritual actions’. We are principally interested in the types of duties that Mīmāṃsā has to contend with, but in order to discuss those types of duties, we must first discuss the types of ritual actions which are involved in Vedic duties.

Conceptually, Mīmāṃsā authors distinguish between a main sacrifice (for instance, the full- and new-moon sacrifices, the dārsāpūrṇamāsa) and each rite constituting it (for instance, an offering to the god Agni, one to the gods Agni and Soma jointly, one to the god Indra within the full- and new-moon sacrifices). They further distinguish between a rite (e.g., the offering of a rice-cake to Agni), and the actions constituting it (e.g., grinding the rice and sifting the flour), which can, again, entail distinct activities (such as putting the rice-grains in the mill and pressing one stone against the other). For the sake of the present article we do not have to analyze all levels of this hierarchy of actions; it is enough to distinguish between the (main) sacrifices and all the actions constituting them (from rites to specific activities). This distinction is at the basis of the differentiation between elective, occasional and fixed sacrifices (see below, sections 2.1.1, 2.1.2 and 2.1.4). We will say that a sacrifice is a primary ritual action and that all its components are subsidiary ritual actions.

2.1 Description of the phenomena

2.1.1 Types of sacrifices and ritual actions

On a practical level, Mīmāṃsā operates with a threefold distinction among types of sacrifices:

1. Fixed sacrifices (nityakarman): Those which one is obligated to perform recurrently throughout one’s life.
   - ‘As long as one lives, one must perform the agniḥotra sacrifice.’ (This is explicitly said to be obligatory for the duration of one’s life.)
   - ‘One should worship at dawn, noon, and twilight.’ (The thrice-daily worship, sandhyāvandana, is also a lifelong duty.)

2. Occasional sacrifices (naimittikakarman): Those which one is obligated to perform when a given condition (the nīmitta ‘occasion’ of the sacrifice) is met. Occasions are specific events happening in one’s life and triggering the performance of a given sacrifice and can vary from the birth of a son to the fact of having broken a vessel during a given sacrifice. The Sanskrit word nīmitta ‘occasion’ can also be translated as ‘cause’ and it evokes the one-to-one relation linking an occasion and the thing it occasions (called naimittika), in this case, the occasion and the sacrifice.
   - ‘On the birth of a son, perform the jāteṣṭi sacrifice.’
   - ‘On the occasion of a solar eclipse, perform the grahaṇaśrāddha.’

Note that occasional sacrifices and fixed sacrifices are very similar, and often there is genuine disagreement about whether a particular sacrifice fits into one or the other category. The essential difference seems to be the fixity or regularity of the occasion upon which one is obligated to perform the sacrifice. Typically, fixed sacrifices have to be performed every day, while occasional sacrifices have to be performed only when something ‘comes up’. However, sacrifices which take place at a specified time in the calendar year (such as the dārsāpūrṇamāsa sacrifices or the jyotiṣṭoma sacrifice) are predictable (unlike the prototypical occasional sacrifices) despite not recurring every day (unlike the prototypical fixed sacrifices).

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6 See, for instance, Exegetic Aphorisms 5.2.6 for a discussion of how each action entails several activities.
According to Mīmāṃsā authors, every sacrifice is said to have some result (phala). We emphasize that the threefold classification occurs on a practical level because its logical structure and implications were never systematically worked out in early Mīmāṃsā. In other words, it is assumed by the authors, as if everyone were familiar with it from other sources, rather than presented and analyzed explicitly. This classification does, however, have important consequences, for example, in determining how stringently the injunctions for performing the sacrifice are to be followed, as well as for understanding the desirable and undesirable results that attach to the performance or non-performance of the sacrifice.

2.1.2 The duty to perform fixed and occasional sacrifices

In the case of fixed and occasional sacrifices, Mīmāṃsakas since the time of Jaimini maintain that the non-performance of the sacrifice is a fault (doṣa). It seems, then, that a fixed sacrifice is strictly obligatory, in the sense that an eligible performer of the sacrifice omits its performance at his peril.

This can be taken to mean that negative consequences attend on the non-performance of a fixed sacrifice:

‘Indeed, this person who, though being a performer of the full- and new-moon sacrifices, omits either the full-moon or the new-moon sacrifice, is cut off from heaven.’

In this case, however, the negative consequences could be interpreted in two slightly different ways: either simply as the absence of a desired result (which is explicitly stated in this case to be heaven, i.e., happiness), or as something that is negative in itself (which may be implied, supposing that the absence of happiness implies the presence of pain, grief, suffering, and so on). The second case would mean that omitting the performance of a fixed or occasional sacrifice would lead to a sanction. Prior to the new phase of Mīmāṃsā associated with Kumārila, Prabhākara and Maṇḍana in the sixth to eighth centuries, there seems to have been very little reflection on the difference between ‘absence of happiness’ and ‘presence of unhappiness’.

2.1.3 General features of sacrifices: results (phala) and eligibility (adhikāra)

According to Mīmāṃsā authors, every sacrifice is said to have some result (phala): for elective sacrifices, this principle is straightforward, since they are defined by an orientation towards a specific desired result; for fixed (and occasional) sacrifices, Mīmāṃsakas maintain that the result is ‘heaven’ (svarga), which they define as felicity (prīti). The desire for heaven is always present in all beings and the so-called ‘Viśvajit Principle’ guarantees that heaven is the result of any sacrifice in which no other result is explicitly mentioned.

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7 lādakarmāṇī ca doṣas tasnāt tato viśeṣaḥ syāt pradhānenaṁbhisanapbandhāt (Exegetic Aphorisms 6.3.3), see fn. 15.
8 Śāhara quotes this passage from a Vedic texts, namely Taśtīrīya Saṁhitā 2.2.5.4, in his commentary on Exegetic Aphorisms 6.3.3.
9 Heaven is explicitly equated to happiness, e.g., in Commentary on Exegetic Aphorisms 6.1.1 (prītisādhane svargaśabda iti) and 6.1.2 (prītī śvarga iti).
10 However, one might try to locate some indirect hints already in Śāhara’s discussions of such sacrifices. For instance, in his commentary on Exegetic Aphorisms 2.4. adiśkarāṇa 1, he discussed the issue of whether the duty to perform the agnihotra sacrifice ‘as long as one is alive’ means that there is a single performance of the agnihotra, repeated through every day of one’s life, or whether an agnihotra is completed every single day and then performed again on the next day. The two alternatives have some bearing on the issue of whether omitting the agnihotra entails the lack of a positive result or an additional sanction. In fact, if there is only a single performance of the agnihotra, lasting throughout one’s whole life, happiness as its result can only come at a successive time, i.e., in the next life, and the absence of this result cannot be the reason for one’s unhappiness during one’s life. If, by contrast, each agnihotra is completed on a given day and delivers its result on a daily basis, then this could be enough to motivate one to refrain from omitting even a single performance.
11 Early Mīmāṃsā authors mention the opinion of an inner-Mīmāṃsā opponent, called Bādari (of whom no works are extant) who claims that the result plays no role at all in sacrifices, but this point of view is explicitly refuted by all later authors. Natarajan 1995 maintains that there is a split among Mīmāṃsakas regarding the connection of each of these types of sacrifice, but the position she ascribes to Prabhākara, namely, that there is no result for fixed and occasional sacrifices, seems closer to the position of Bādari than to the one of Prabhākara. If we exclude Bādari, then all Mīmāṃsakas agree that every sacrifice leads to a result.
Secondly, every sacrifice is characterized by an eligibility (adhikāra). The eligibility identifies the person who will possess the result of the sacrifice. This is particularly relevant in the context of Vedic sacrifices, since they always include many performers. The adhikārin is the one who decides to perform the sacrifice (and pays several Brāhmaṇas to perform it) and the sacrifice’s result will accrue to him only. Thus, being eligible means being both enjoined to perform the sacrifice and entitled to its result; the former implies that one can actually hear a Vedic injunction and understand oneself as the addressee, which, in general, implies that one belongs to one of the social groups that is traditionally expected to study the Vedas. Being eligible further implies having enough material wealth to organize the performance of a sacrifice: eligibility thus always includes ability, and so it incorporates a version of the ‘ought entails can’ thesis, according to which an injunction to perform a certain sacrifice applies only to a person that is able, in principle, to complete the performance of that sacrifice. For instance, a lame person who desires cattle and hears the Vedic injunction ‘The one who desires cattle should sacrifice with the citrā sacrifice’ is, notwithstanding her desires, not the addressee of the injunction because she could not be able to carry out the sacrificial actions required. The negative consequences that, as noted above (section 2.1.1), would generally follow from non-performance of a sacrifice only apply to an eligible sacrificer’s non-performance and negative consequences could follow if she does perform it. By contrast, a person who has the physical (see Exegetic Aphorisms 6.1.42) and economical ability (see Exegetic Aphorisms 6.7.18–20) to perform a given sacrifice and desires its output is the addressee of the relevant injunction, independently of her decision to carry it out or not. Accordingly, eligibility applies to a person for a longer period of time than will, desire, or inclination and is defined, partly, by the wherewithal needed to successfully perform a given sacrifice. Eligibility is not about possessing a given ability temporarily, but about one’s overall condition as a human being. For instance, if a blind person were to receive once in a month and for one day only a digital visor enabling her to see, she would still lack the adhikāra for performing a sacrifice lasting longer than the day in which she can see. A person who should receive a huge amount of money for only one day would not qualify as wealthy enough to perform a complex sacrifice and so on. Symmetrically, if one were to temporarily lose one’s ability to see (e.g., because one has looked directly into the sun), or if one had run out of a given ritual substance, one would not lose one’s adhikāra—the loss of adhikāra is sometimes discussed in the texts, but only as a consequence of some traumatic experience (e.g., a major violation of dharma rules).

2.1.4 The duty to perform elective sacrifices

What makes a sacrifice ‘elective’ rather than fixed or occasional is the presence of a desire for a specific result (as distinguished from the desire for happiness, which is always present) and the fact that there are no negative consequences in case one does not perform it, apart from not getting the intended result. Furthermore, the deontic strength of injunctions to perform elective sacrifices seems weaker: for example, there is an injunction that says ‘one who desires cattle should perform the citrā sacrifice.’ Suppose that you desire cattle and satisfy the eligibility requirement: are you under an obligation to perform the citrā sacrifice? Most Mimāṃsakas would say that you are not, i.e., that even one who desires the specified result can omit the performance of the sacrifice without any adverse consequences, apart from the absence of the desired result. This could mean that fixed and occasional sacrifices differ from elective sacrifices in the fact that their result is something one cannot live without (i.e., happiness), so that its absence is the sumnum malum.

Under these circumstances, it is unclear whether we should speak of an ‘obligation’ to perform elective sacrifices at all. On the one hand, there is a strong presumption in Mimāṃsā that all Vedic injunctions operate in the same way, although there is considerable debate about how they operate. This means, in particular, that in the Vedas injunctions to perform elective sacrifices have the same linguistic presentation as injunctions to perform fixed and occasional sacrifices. Thus, if we interpreted injunctions to perform fixed and occasional sacrifices as obligations relying only on their linguistic presentation, we should, in principle, do the same with injunctions to perform elective sacrifices. On the other hand, in Euro-American deontic theories, obligations are often considered as ‘duals’ of permissions, so that the performance of an action is obligatory if and only if its non-performance is not permitted. It is perhaps worth noting that this is not necessarily the case with all obligations in all cultures. One is, for instance, immediately reminded of the contrast between the law and grace in Paul, or of the paradox of the ‘infinite responsibility’ discussed by Emmanuel Levinas. In an Indian context, one may evoke the Viśiṣṭādvaita Vedānta analysis of one’s helplessness in regard to moral laws which could never be fulfilled.

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13 See Mabbott 1966 for a concise survey of traditional deontic theories in Euro-American philosophy and a discussion of the various notions of ‘duty’ employed in these theories.
have seen that the non-performance of elective sacrifices is permitted, though not recommended. Thus, it seems plausible to conceive injunctions to perform elective sacrifices as recommendations.

A further complication is that once one has undertaken the performance of an elective sacrifice, then she has a duty to carry out the sacrifice completely. Mīmāṃsā authors say in this connection that an elective sacrifice is undertaken because of a desire; but once it has been undertaken, the undertaking itself (ārambha) serves as the occasion (nimitta) for the completion of the elective sacrifice. Śabara suggests (in Commentary on Exegetic Aphorisms 6.1.15) that there is also a social sanction if you start performing an elective sacrifice without being able to complete it, in the sense that people will think less of you.

6.1.5 The ‘good enough’ principle (yathāśakti-nyāya)

Regarding the subsidiaries of fixed sacrifices, but not of elective sacrifices, it is said that one must perform them ‘insofar as one is able’ (yathāśakti). If you undertake a fixed sacrifice like the agnihotra sacrifice, then you do not have a duty to ‘dot the is and cross the ts.’ That is to say, if the Vedas enjoin certain actions as subsidiaries of the agnihotra, and you are not in a position to perform those actions, then you may simply omit them and the injunction to perform the agnihotra will nevertheless be taken as fulfilled. This represents what we can call the ‘good enough’ principle.

A few more words are needed in order to show how the ‘good enough’ principle relates to the notion of eligibility (adhikāra, see section 2.1.3). As we said, eligibility postulates that one must in theory be able to perform a sacrifice in order to be the addressee of the relevant injunction. If one is a priori unable to perform the sacrifice, for instance, because one cannot walk or cannot afford the purchase of the sacrificial substances, one is not the addressee of the sacrificial prescriptions and is therefore not under their obligations. By contrast, if one is a priori able to perform the sacrifice but concretely unable to perform a specific subsidiary act (for instance, because one does not have a given sacrificial substance available), one can perform the whole sacrifice according to the ‘good enough’ principle without violating her obligation.

How should we understand, exactly, the distinction between a priori impossibility and concrete impossibility? It is not a distinction between a deficit regarding the person and a deficit regarding her wealth (so that one could say that the adhikāra regards the person and the yathāśakti nyāya her possessions) for a person who cannot have enough substances for the performance of a sacrifice (e.g., because she cannot own wealth) lacks the relevant adhikāra (see Exegetic Aphorisms, chapter 6.1, see also above, section 2.1.3). Rather, the distinction involves a general possibility and a concrete condition. A person who is in general physically and economically able to perform a sacrifice has the relevant adhikāra. Her lacking a concrete sacrificial ingredient at a given moment of time, while nothing precludes that she might acquire it at a later moment of time, does not hinder the fact that she has the adhikāra.

In contrast, regarding elective sacrifices, it is said that one must perform them by completing all their subsidiaries exactly as prescribed in the Vedas. In other words, if one does not perform the subsidiary actions of a given sacrifice precisely as they are enjoined in the Vedic text, her performance does not count as a satisfactory performance of the enjoined sacrifice.

2.1.6 Discussion

Let us review what we have seen so far, which represents the system of Common Mīmāṃsā: The standard classification of sacrifices distinguishes three principal types, namely, fixed, occasional, and elective. We can abstract at least two kinds of deontic properties that apply to primary actions:

- obligation: You omit the enjoined action at your peril. This applies to fixed and occasional sacrifices.
- recommendation: You may omit the enjoined action without any adverse consequences. This applies to elective sacrifices, but only before one has started them; if one has undertaken an elective sacrifice, one fails to complete it at one’s peril.

Furthermore, we can distinguish between two ways in which one has to perform subsidiary actions:

15 Exegetic Aphorisms 6.3, adhikāraṇa 1. See especially Exegetic Aphorisms 6.3.3 (tadakarmāṇi ca doṣas tasmāt tato viśeṣaḥ syat prabhāvoddhīsanbandhat ‘If you do not perform it (the principal sacrifice) there is a fault, therefore there is a distinction between the [principal sacrifice] [and the auxiliaries], since the [fault] is mentioned only in connection with the principal sacrifice’).
16 See Exegetic Aphorisms 6.3, adhikāraṇa 2.
– ‘exactly as prescribed in the texts’: this applies to the subsidiaries of elective sacrifices and means that their performance does not count unless it is done precisely as the Vedas prescribe.
– ‘as much as possible’: this applies to the subsidiaries of fixed and occasional sacrifices and means that their performance counts even if it is not done precisely as the Vedas prescribe, as long as it is done to the best of the sacrificer’s ability.

In a sense, there is a kind of deontic reversal when moving from primary actions (i.e., sacrifices) to their subsidiaries. If a sacrifice is obligitory, then its subsidiaries are to be performed as much as one can (insofar as the ‘good enough’ principle applies) and so it is permitted that some of them are not performed or are simplified. By contrast, if a sacrifice is recommended, then its subsidiaries are to be performed exactly as prescribed (insofar as the ‘good enough’ principle does not apply), so it is not permitted to simplify their performance.

3 Formalizing the Mīmāṃsa theory

3.1 The system $S_0$

The distinction among types of Vedic duties discussed by Mīmāṃsa authors and illustrated in section 2.1 relies on the interplay between certain deontic notions and certain types of circumstance; in this section we move from an informal to a formal setting and provide a logical analysis of the Mīmāṃsa theory. We start by observing that standard languages of propositional logic are not adequate to capture basic differences among ritual actions, since this requires the possibility of analyzing the internal structure of propositions. For instance, in standard propositional languages it is not possible to define, in general terms, the properties of sacrifices that should be performed in circumstances of different kind (e.g. when a relevant occasion takes place or a relevant desire arises) or the relation between a sacrifice and its subsidiary actions. Here we propose to adopt a slightly richer language, hereafter simply called $L$, which simulates some aspects of first-order languages while remaining closely related to propositional ones, since it does not involve any form of quantification. Our choice is motivated by the aim of keeping the logical analysis as simple as possible.

The language $L$ is built ad hoc to represent the theory of Vedic duties in early Mīmāṃsa and includes four categories of individual constants: sacrifice constants (which denote entities like ‘the citrā sacrifice’ and ‘the full-moon sacrifice’) action constants (which denote entities like ‘grinding the rice’ and ‘sifting the flour’), outcome constants (which denote entities like ‘cattle’ and ‘happiness’) and event constants (which denote entities like ‘the birth of a son’ and ‘a solar eclipse’). These constants are used to build atomic formulas of some specified kind, thanks to the presence of a small set of predicate symbols, which express fundamental notions in the Mīmāṃsa analysis. Furthermore, it includes a propositional constant associated with the notion of eligibility, boolean operators and intensional operators for strong/weak duties. The full list of primitive symbols in $L$ is the following:

- a countable set of individual constants for sacrifices ($\text{sacrifice constants}$) $\text{SAC} = \{s_1, s_2, s_3, \ldots\}$;
- a countable set of individual constants for actions ($\text{action constants}$) $\text{ACT} = \{a_1, a_2, a_3, \ldots\}$;
- a countable set of individual constants for events ($\text{event constants}$) $\text{EVT} = \{e_1, e_2, e_3, \ldots\}$;
- a countable set of individual constants for outcomes ($\text{outcome constants}$) $\text{OUT} = \{o_1, o_2, o_3, \ldots\}$;
- a propositional constant (eligibility constant) $e$;
- the boolean connectives $\neg$ (negation) and $\rightarrow$ (material implication);
- the intensional operators $O$ (strong duty, or obligation) and $R$ (weak duty, or recommendation);
- the unary predicate symbol $\text{und}$ (for ‘is undertaken’) applied to elements of the set $\text{SAC}$;
- the unary predicate symbols $\text{cap}$ (for ‘is performed exactly as prescribed’), $\text{yathāyāya}$ and $\text{amp}$ (for ‘is performed as much as possible’, $\text{yathūsāktii}$) applied to elements of the set $\text{ACT}$ (see section 2.1.5 for a discussion of the corresponding Sanskrit terms);
- the unary predicate symbol $\text{des}$ (for ‘... is desired’) applied to elements of $\text{OUT}$;
- the unary predicate symbol $\text{tpl}$ (for ‘... takes place’) applied to elements of $\text{EVT}$;
- the binary predicate symbol $\text{sub}$ (for ‘... is a subsidiary of ...’) applied to elements of the set $\text{ACT} \times \text{SAC}$ (i.e., the cartesian product of $\text{ACT}$ and $\text{SAC}$).

\footnote{This means that the binary predicate symbol $\text{sub}$ only defines the relation linking a subsidiary to a main sacrifice. It does not define the relation holding between a subsidiary and its own further subsidiaries (e.g., the relation holding between ‘offering a rice-cake’ and ‘grinding the rice’ or ‘baking the cake’). We decided to focus on the first distinction since this alone has consequences for the categorisation of sacrifices. This is also the reason why we introduced a special category for sacrifices, namely $\text{SAC}$.}
The exhaustive meaning of these symbols will be clarified below. The boolean connectives \( \land \) (conjunction), \( \lor \) (inclusive disjunction) and \( \equiv \) (material equivalence), are defined in the usual way in terms of the primitive ones. It is convenient to consider also the set of propositional atoms in \( \mathcal{L}, \text{ATOM} \), which is the smallest set described by the following clauses:

- for any \( s_1 \in \text{SAC} \), \( \text{und}(s_1) \in \text{ATOM} \);
- for any \( a_i \in \text{ACT} \), \( \text{cap}(a_i) \), \( \text{amp}(a_i) \in \text{ATOM} \);
- for any \( e_i \in \text{EVT} \), \( \text{tpl}(e_i) \in \text{ATOM} \);
- for any \( o_i \in \text{OUT} \), \( \text{des}(o_i) \in \text{ATOM} \);
- for any \( a_i \in \text{ACT} \) and \( s_j \in \text{SAC} \), \( \text{sub}(a_i, s_j) \in \text{ATOM} \);
- \( \epsilon \in \text{ATOM} \).

An arbitrary member of \( \text{ATOM} \) will be denoted by \( \epsilon \). Propositional atoms have to be read as follows: \( \text{und}(s_i) \) means that the sacrifice described by \( s_i \) (e.g., the full-moon sacrifice) is undertaken; \( \text{cap}(a_i) \) that the action described by \( a_i \) (e.g., grinding the rice) is carried out exactly as prescribed and \( \text{amp}(a_i) \) that the same action is performed as much as possible (i.e., according to one’s possibility); \( \text{tpl}(e_i) \) means that the event described by \( e_i \) (e.g., the full moon) takes place and \( \text{des}(o_i) \) means that the outcome described by \( o_i \) (e.g., cattle) is desired; \( \text{sub}(a_i, s_j) \) means that the action described by \( a_i \) is a subsidiary of the sacrifice described by \( s_j \) (e.g., offering a rice-cake to Agni is a subsidiary of the full-moon sacrifice). Finally, we can read the propositional constant \( \epsilon \) as ‘minimal requirements to perform sacrifices are satisfied’, so this constant represents eligibility.\(^{18}\)

The set of well-formed formulas (hereafter, wffs) of \( \mathcal{L} \) is the smallest set satisfying the clauses below:\(^{19}\)

- \( \epsilon \) is a wff, for any \( \epsilon \in \text{ATOM} \);
- if \( \phi \) is a wff, then so are \( \neg(\phi) \), \( O(\phi) \) and \( R(\phi) \);
- if \( \phi \) and \( \psi \) are wffs, then so is \( (\phi \rightarrow \psi) \).

Round brackets that should apply to the arguments of boolean and intensional operators are omitted in formulas if there is no risk of ambiguity; for instance, \( (\phi \rightarrow (\psi) \) can be rewritten as \( \phi \rightarrow \psi \). We use the notion of main operator in a formula in the usual way; for instance, in a formula of kind \( \phi \rightarrow (\psi \rightarrow \chi) \) we say that the first occurrence of \( \rightarrow \) is the main material implication. We propose to read \( O\phi \) as ‘in every situation in which strong Vedic duties are observed \( \phi \) is the case’ and \( R\phi \) as ‘in every situation in which weak Vedic duties are observed \( \phi \) is the case’. The difference between strong and weak Vedic duties resembles the difference between obligations and recommendations, as it was pointed out in sections 2.1.2 and 2.1.4: thus, another possible reading of \( O\phi \) and \( R\phi \) is, respectively, ‘it is obligatory, according to the Vedas, that \( \phi \)’ and ‘it is recommended, according to the Vedas, that \( \phi \)’.

We stipulate the following definitions (for \( s_i \in \text{SAC} \)), which capture the main properties of the three types of sacrifice:

- \( \text{fixed}(s_i) \equiv R(\epsilon \rightarrow \text{und}(s_i)) \);
- \( \text{optional}(s_i) \equiv \epsilon \land \text{tpl}(e_n) \rightarrow \text{und}(s_i) \);
- \( \text{elective}(s_i) \equiv \epsilon \land \text{des}(o_n) \rightarrow \text{und}(s_i) \).

Thus, a sacrifice is fixed if and only if (hereafter, iff) it is obligatory to undertake it in any circumstance in which eligibility is met; a sacrifice is occasional iff it is obligatory to undertake it whenever eligibility is met and the relevant occasion takes place; finally, a sacrifice is elective iff it is recommended to undertake it whenever eligibility is met and the relevant desire arises.

We will now build a system \( S_0 \) over the language \( \mathcal{L} \) in order to be able to represent generic patterns of reasoning in terms of our formalization of the Mīmāṃsā theory of Vedic injunctions. We will proceed in a series of steps, illustrating some plausible principles that can be taken into account in the axiomatic

\(^{18}\) See the discussion on the notion of eligibility in section 2.1.3. For the use of propositional constants in a deontic setting, see Anderson 1956 and Åqvist 1987. Propositional constants allow one to codify in a formal language certain states-of-affairs which are particularly relevant for deontic reasoning; in our case, the notion of eligibility. Indeed, we will see that the constant \( \epsilon \) restricts the set of situations in which certain ritual actions have to be performed.

\(^{19}\) In a sense, the language \( \mathcal{L} \) captures two different intuitions that are often compared in deontic logic. One intuition, originated with von Wright 1951, is that actions have a special status in deontic reasoning; this intuition, which leads to an analysis of what ‘ought to do’ is here captured by making explicit reference to actions via individual constants. The other intuition, originated with Anderson 1956, is that sometimes deontic reasoning involves relations among states-of-affairs (e.g., ‘if there are criminals, there ought to be sanctions’); this intuition, which leads to an analysis of what ‘ought to be the case’, is here captured by the possibility of applying boolean and intensional operators to arbitrary formulas, rather than formulas involving actions alone.
basis of \( S_0 \). First of all, we want \( S_0 \) to be an extension of the classical propositional calculus\(^{20}\) so we assume what follows (where the expression \( \vdash_{S_0} \phi \) means that \( \phi \) is a theorem of \( S_0 \) and the expression \( \phi_1, \ldots, \phi_n \vdash_{S_0} \phi_{n+1} \) means that \( \phi_{n+1} \) is derivable in \( S_0 \) from the list of assumptions \( \phi_1, \ldots, \phi_n \): \[ \vdash_{S_0} \phi, \text{ for any propositional tautology } \phi \] (1) \[ \phi, \phi \rightarrow \psi \vdash_{S_0} \psi \] (2) Then, we add a series of axiom-schemata (hereafter, also called axioms or schemata, for the sake of brevity) concerning the intuitive relation between some predicate symbols in \( L \).\(^{21}\) These axiom-schemata allow us to make universal claims with reference to certain classes of individual constants. The first schema (where \( a_i \) is any constant in \( ACT \)) concerns the relation between \( cap \) and \( amp \): \[ eap(a_i) \rightarrow amp(a_i) \] (3) Axiom-schema (3) means that whenever an action is completed, then that action is also performed as much as possible. We also need some principles concerning the interaction between fixed, occasional, elective sacrifices and their subsidiaries. Consider the following ones: \[ (\text{fixed}(s_i) \land \text{sub}(a_j, s_i)) \rightarrow \mathcal{O}(\mathcal{e} \rightarrow (\text{und}(s_i) \rightarrow \text{amp}(a_j))) \] (4) \[ (\text{occasional}(s_i) \land \text{sub}(a_j, s_i)) \rightarrow \mathcal{O}((\mathcal{e} \land \text{tp}(e_n)) \rightarrow (\text{und}(s_i) \rightarrow \text{amp}(a_j))) \] (5) \[ (\text{elective}(s_i) \land \text{sub}(a_j, s_i)) \rightarrow \mathcal{O}((\mathcal{e} \land \text{des}(o_n)) \rightarrow (\text{und}(s_i) \rightarrow \text{eap}(a_j))) \] (6) Axiom-schema (4) says that, obligatorily, whenever eligibility is met (\( \mathcal{e} \)), if a fixed sacrifice (\( s_i \)) is undertaken, then any of its subsidiaries (\( a_j \)) is performed as much as possible. Axiom-schema (5) says that, obligatorily, whenever eligibility is met (\( \mathcal{e} \)) and a certain occasion (\( e_n \)) takes place, if a relevant occasional sacrifice (\( s_i \)) is undertaken, then any of its subsidiaries (\( a_j \)) is performed as much as possible. Axiom-schema (6) says that, obligatorily, whenever eligibility is met (\( \mathcal{e} \)) and a certain outcome (\( o_n \)) is desired, if a relevant elective sacrifice (\( s_i \)) is undertaken, then any of its subsidiaries (\( a_j \)) is performed exactly as prescribed. These axioms represent the ‘deontic reversal’ mentioned in section 2.1.6: on the one hand, fixed sacrifices represent duties in a strong sense (or obligations, \( \mathcal{O} \)), while their subsidiaries have to be performed only as much as possible (\( \text{amp} \)); on the other hand, elective sacrifices represent duties in a weak sense (or recommendations, \( \mathcal{R} \)), while their subsidiaries, once the main action is undertaken, have to be carried out precisely as prescribed (\( \text{eap} \)).

Furthermore, we need to add axioms for our operators \( \mathcal{O} \) and \( \mathcal{R} \). In the case of \( \mathcal{O} \), a possible solution consists in extracting the intended axioms and rules from a preliminary system to represent deontic reasoning in Mīmāṃsa, developed in Ciabattoni et al. 2015; such system, called \( \text{bMDL} \) (basic Mīmāṃsa Deontic Logic), is based on a propositional language with both alethic and deontic modalities. The result of this ‘extraction procedure’ is the following list of principles (see the Appendix for the detailed proof and for an explanation of the sense in which \( \text{bMDL} \) can be said to be a system inspired by the Mīmāṃsa school):

\[ \phi \equiv \psi \quad \mathcal{O}\phi \equiv \mathcal{O}\psi \] (7) \[ \mathcal{O}(\phi \land \psi) \rightarrow (\mathcal{O}\phi \land \mathcal{O}\psi) \] (8) \[ \neg(\mathcal{O}\phi \land \mathcal{O}\neg\phi) \] (9) The principle (7) says that if the states-of-affairs described by \( \phi \) and \( \psi \) are logically equivalent, then either both or neither ought to be the case (more precisely, according to our specific reading of \( \mathcal{O} \), either both or neither hold in all situations in which strong Vedic duties are observed). The principle (8) says that if the states-of-affairs described by \( \phi \) and \( \psi \) jointly form a state-of-affairs which ought to be the

\(^{20}\) The legitimacy of this move is explained in Freschi et al. 2017. In fact, a Mīmāṃsa embedded in Jayanta’s Nyāya-śāstra explains how, when a contradiction is considered, denying one alternative makes the other true. This implies the legitimacy of reductio ad absurdum, thus justifying the use of classical logic.

\(^{21}\) In our presentation of axiom-schemata there is some abuse of notation; for instance, in the case of (3) we should employ a symbol different from \( a_i \), which would stand for an arbitrary element of \( ACT \) (namely a metalinguistic symbol); however, in our opinion the notation adopted here is more accessible for non-logicians and we will try to avoid any risk of confusion by supporting the presentation of various axiom-schemata with some informal comments.
case, then each of them ought to be the case. Finally, (9) says that it is not the case that two conflicting states-of-affairs ought to be the case.\footnote{Here the term ‘situation’ is used in an informal way. In section 3.2 a situation will represent a completely determined and exhaustive set of states-of-affairs, namely what is usually called a (possible) ‘world’ in the literature on intensional logic. For further philosophical analysis of the relation between worlds and states-of-affairs, see \textit{Adams 1974}.}

In the case of $\mathcal{R}$, instead, a different set of logical principles is required; in particular $\neg(\mathcal{R}\phi \land \mathcal{R}\neg\phi)$ seems too strong, since $\mathcal{R}$ expresses a deontic property which is closer to recommendation than obligation and there are examples of conflicting recommendations in the Vedas, such as $\mathcal{R}\phi$ and $\mathcal{R}\psi$, where $\psi$ entails $\neg\phi$. For instance, there are two different Vedic prescriptions concerning the desire of rain. One prescription says that if a person desires rain, he or she should perform the $k\ddot{a}r\ddot{a}r\ddot{i}$ sacrifice; the other prescription says that if a person desires rain, he or she should perform the \textit{twelve-nights} sacrifice (\textit{Kaus\u{i}ka S\u{u}tra} 5.5.41.1); however, a person cannot perform both sacrifices at the same time (and probably would not perform them in succession, given that one of the two sacrifices is enough to get the intended result).

Accordingly, we propose to replace $\neg(\mathcal{R}\phi \land \mathcal{R}\neg\phi)$ with $\neg\mathcal{R}(\phi \land \neg\phi)$, which guarantees, at least, that there are no self-contradictory recommendations. Thus, we take the following set of principles for $\mathcal{R}$:

\begin{align}
\phi &\equiv \psi \\
\mathcal{R}\phi &\equiv \mathcal{R}\psi
\end{align}

$\mathcal{R}(\phi \land \psi) \rightarrow (\mathcal{R}\phi \land \mathcal{R}\psi)$

$\neg\mathcal{R}(\phi \land \neg\phi)$

The reading of (10), (11) and (12) can be inferred from our considerations above and their relations with (7), (8) and (9). The axiomatic basis of the system $S_0$ is finally defined by the list of principles (1)-(12).

Notice that in $S_0$ we do not impose any connection between $\mathcal{O}$ and $\mathcal{R}$, since, according to our reading of the two operators, $O\phi$ simply means that $\phi$ represents a strong duty in the Vedas and $R\phi$ that $\phi$ represents a weak duty in the Vedas. On the one hand, relying on the parallelism mentioned above between strong duties and obligations and on the one between weak duties and recommendations (which inspired a simpler reading for $\mathcal{O}$ and $\mathcal{R}$), one could add the further principle that whatever is obligatory is also recommended:

$O\phi \rightarrow R\phi$

On the other hand, sometimes Mīmāṃsā authors encountering Vedic statements which seem to prescribe as an elective sacrifice what is already known to be a fixed sacrifice read these statements as prescribing a different sacrifice, bearing by chance the same name as the fixed sacrifice (see \textit{Exegetic Aphorisms} 2.3.24–25). This leads to a completely different idea of the relation between strong duties and weak duties, namely that they are mutually exclusive; we can formalize this idea by means of the principle below:

$O\phi \rightarrow \neg R\phi$

In what follows we will focus on the semantic characterization of the basic system $S_0$, in which neither of the schemata (13) and (14) is taken into account. However, if we call $S_1$ and $S_2$ the systems obtained by respectively adding (13) and (14) to the axiomatic basis of $S_0$, we will see that both $S_1$ and $S_2$ are consistent systems.

3.2 Semantic characterization of $S_0$

We introduce now a formal semantics for the system $S_0$. We interpret the language $\mathcal{L}$ in \textit{neighborhood frames}, which are here structures of kind $\mathcal{F} = \langle \mathcal{W}, \mathcal{W}_e, N^+, N^- \rangle$. $\mathcal{W} = \{w_1, w_2, w_3, \ldots \}$ (also $w, v, u, \ldots$) is a set of possible situations (or worlds) and $\mathcal{W}_e \subseteq \mathcal{W}$ is the set of situations in which the eligibility conditions are met. $N^+ : \mathcal{W} \rightarrow \{\varphi(\varphi(W))\}$ is said to be a \textit{neighborhood function}; for any $w \in \mathcal{W}$, $N^+(w) \subseteq \varphi(W)$ is said to be the $N^+$-sphere of $w$. Similar definitions can be used for $N^-$. Informally, $N^+(w)$ and $N^-(w)$ can be respectively taken as representing the set of strong Vedic duties (obligations) and the set of weak Vedic duties (recommendations) which apply to the situation $w$.

A model over a neighborhood frame, also called \textit{neighborhood model}, is a structure of kind $\mathfrak{M} = \langle \mathcal{F}, V \rangle$, where $\mathcal{F}$ is the underlying frame and $V : ATOM \rightarrow \varphi(W)$ is said to be a \textit{valuation function}. For any
at ∈ ATOM, V(ata) ⊆ W is said to be the valuation of at in M. The valuation function satisfies the minimal property \( V(e) = W_e \), which ensures that in every model the set of situations in which eligibility is met corresponds to the set \( W_e \). Models can be denoted also by pointing out elements of the underlying frame, as in \( M = \langle W, W_e, N^+, N^-, V \rangle \). Since in the language \( L \) quantification over individuals is not available and individual constants are interpreted in the same way across worlds and models, we do not need to add domains of individual entities to our frames and models. This fact provides a semantic justification to our claim that \( L \) remains closely related to propositional languages (while allowing one to analyze the internal structure of propositions).

Given a model \( M \), we say that all strong (respectively, weak) Vedic duties are observed in a situation \( w \) iff \( w \in X \) whenever \( X \in N^+(w) \) (respectively, \( X \in N^-(w) \)). Indeed, this corresponds with the fact that \( w \) verifies all instances of the schema \( O \phi \to \phi \) (respectively, \( R \phi \to \phi \)). Notice that, in principle, different situations can be associated with different duties.

Formulas of \( L \) are evaluated with reference to a world in a model. Truth-conditions for formulas are defined below, where \( M \) and \( w \) are, respectively, an arbitrary model and an arbitrary world in it (given a formula \( \phi \in L \), the expression \( M, w \models \phi \) means that \( \phi \) is true at \( w \) in \( M \), whereas the expression \( M, w \not\models \phi \) means that \( \phi \) is false at \( w \) in \( M \)):

- for any \( at \in ATOM \), \( M, w \models at \) iff \( w \in V(at) \);
- \( M, w \models \psi \to \chi \) iff \( M, w \not\models \psi \); \( M, w \not\models \phi \) or \( M, w \models \psi \);
- \( M, w \not\models O \phi \) iff \( \{v \in W : M, v \models \phi \} \in N^+(w) \), where \( \{v \in W : M, v \models \phi \} \in N^+(w) \), where \( \phi \in \mathcal{L} \);
- \( M, w \not\models R \phi \) iff \( \{v \in W : M, v \models \phi \} \in N^-(w) \);

It is interesting to explore the intuition behind the last two clauses in the list. Indeed, relying on a convention about the interpretation of intensional operators in standard neighborhood models, we can say that in a model any injunction corresponds to the set of situations in which it is fulfilled (notice that this convention is grounded on our reading of \( O \phi \) and \( R \phi \) as ‘in all situations in which strong Vedic duties are observed, \( \phi \) is the case’ and ‘in all situations in which weak Vedic duties are observed, \( \phi \) is the case’).

A formula \( \phi \) is valid in a model \( M \) iff for all \( w \in W \), we have \( M, w \models \phi \); we represent this fact also as \( M \models \phi \). A formula \( \phi \) is valid in a class of models \( C \) iff, for every \( M \in C \), we have \( M \models \phi \); we represent this fact also as \( C \models \phi \). A system \( X \) built over the language \( L \) is sound with respect to (hereafter, w.r.t.) a class of models \( C \) iff every formula which is a theorem of \( X \) is valid in \( C \); \( X \) is complete w.r.t. \( C \) iff every formula which is valid in \( C \) is a theorem of \( X \); \( X \) is characterized by a class of models \( C \) iff it is both sound and complete w.r.t. \( C \). If a system \( X \) is sound w.r.t. a class of models \( C \), we also say that \( C \) is a class of models for \( X \). We can similarly speak of soundness, completeness and characterization of a system \( X \) w.r.t. a single model \( M \). Finally, a system \( X \) is consistent iff there exists some model for \( X \).

We introduce now a specific class of models which will be used for the semantic characterization of \( S_9 \). Let \( K_M \) be the class of neighborhood models \( M = \langle W, W_e, N^+, N^-, V \rangle \) satisfying the following properties:

- \( \text{P1} \) for any \( X, Y \in \varphi(W) \) s.t. \( X \subseteq Y \) and any \( w \in W \), if \( X \in N^+(w) \), then \( Y \in N^+(w) \);
- \( \text{P2} \) for any \( X, Y \in \varphi(W) \) s.t. \( X \subseteq Y \) and any \( w \in W \), if \( X \in N^-(w) \), then \( Y \in N^-(w) \);
- \( \text{P3} \) for any \( X \in \varphi(W) \) and any \( w \in W \), if \( X \in N^+(w) \), then \( X = \{v \in W : v \notin X \} \not\in N^+(w) \);
- \( \text{P4} \) for any \( w \in W, \ 0 \notin N^-(w) \);
- \( \text{P5} \) for any \( a_j \in ACT, V(eap(a_j)) \subseteq V(amp(a_j)) \);
- \( \text{P6} \) for any \( a_j \in ACT, s_i \in SAC \) and \( w \in W \), if \( v \in V(sub(a_j, s_i)) \) and \( \|e \to \text{und}(s_i)\|^{2M} \in N^+(w) \), then \( \|e \to \text{und}(s_i) \to \text{amp}(a_j)\|^{2M} \in N^+(w) \);
- \( \text{P7} \) for any \( a_j \in ACT, s_i \in SAC \) and \( w \in W \), if \( v \in V(sub(a_j, s_i)) \) and \( \|e \land \text{.tpl}(e_n) \to \text{und}(s_i)\|^{2M} \in N^+(w) \), for some \( e_n \in \text{EVT} \), then \( \|e \land \text{.tpl}(e_n) \to \text{und}(s_i) \to \text{amp}(a_j)\|^{2M} \in N^+(w) \);
- \( \text{P8} \) for any \( a_j \in ACT, s_i \in SAC \) and \( w \in W \), if \( v \in V(sub(a_j, s_i)) \) and \( \|e \land \text{des}(a_o) \to \text{und}(s_i)\|^{2M} \in N^-(w) \), for some \( a_o \in \text{OUT} \), then \( \|e \land \text{des}(a_o) \to \text{und}(s_i) \to \text{eap(a_j)}\|^{2M} \in N^-(w) \).

We will prove that the system \( S_9 \) is sound and complete w.r.t. the class \( K_M \). First, we show that \( K_M \) is a non-empty class, namely that there is at least one model satisfying properties P1-P8. We provide a model based on the argument illustrating the duty to perform the citrā sacrifice.

Let us consider the Mimāṃsā analysis of the citrā sacrifice: The citrā sacrifice is an elective sacrifice prescribed for people who desire cattle. One who is eligible (i.e., because he has studied the Vedas, is

23 For a presentation of standard neighborhood frames and models, see Chellas 1980.
physically able to perform a sacrifice and possesses enough wealth to perform it) and desires cattle should perform the citra sacrifice. Since it is an elective sacrifice, one needs to carry out its subsidiaries, such as the offerings of a cake on eight cups to Agni, of a caru on eleven cups to Sarasvatī, of a caru on eight cups to Vāsṭr, of a caru on eight cups to Sarasvatī, and of a cake on eleven cups to Indra, exactly as prescribed in the Vedas. Let s denote the citra sacrifice, o1 denote cattle (as an outcome) and a1, ..., a7 denote, respectively, the offerings to Agni, to Sarasvatī, to Vaṣṭr, to Sarasvatī, and to Indra. We do not need to talk about events in this sacrifice, so we can set EVT = ∅. Consider the model ℳ = ⟨W, Wc, N+, N−, V⟩ s.t. W = {w1, w2, w3}, Wc = {w2, w3}, N+(w1) = N+(w2) = N+(w3) = 0, N+(w1) = {w1, w2, w}, N+(w2) = N+(w3) = 0, V(sub(a1, s1)) = W for 1 ≤ j ≤ 7, V(des(a1)) = V(und(s1)) = {w1, w2, w3}, V(cap(a1)) = V(amp(a1)) = {w2, w3} for 1 ≤ k ≤ 6, V(amp(a2)) = {w3, w} and V(cap(a2)) = {w2, w3}. Properties P1–P4 are clearly valid in ℳ. It is easy to check that property P5 holds as well, given that all ritual actions a1, ..., a7 are performed exactly as prescribed by the Vedas only in situations in which they are performed as much as possible; more precisely, a7 is the only action which is performed as much as possible in a situation, w1, without being performed there exactly as prescribed by the Vedas — for instance, because one offers a nine-cup (rather than an eleven-cup) cake to Indra. The only relevant situation to check the validity of properties P6, P7 and P8 is w1, since N+(w1) = N+(w2) = N+(w3) = 0. In the case of P6, the result follows from the fact that |ϕ → und(s1)||ℳ = 1; w2, w3 /∈ N+(w1). P7 trivially holds since we have EVT = ∅. Finally, in the case of P8, given that |ϕ ∧ des(a1)| → und(s1)||ℳ = |ϕ ∧ des(a1)| → (und(s1) → cap(a1))||ℳ = W for 1 ≤ k ≤ 6 and |ϕ ∧ des(a1)| → (und(s1) → cap(a1))||ℳ = {w1, w2, w3}, then we have both |ϕ ∧ des(a1)| → und(s1)||ℳ ∈ N+(ϕ) and |ϕ ∧ des(a1)| → (und(s1) → cap(a1))||ℳ ∈ N+(ϕ) (for 1 ≤ j ≤ 7), which is enough. Notice that in this model w3 is a situation in which the duty of the citra sacrifice is not observed, since one undertakes the sacrifice without performing all subsidiaries (in particular, the offering to Indra) exactly as prescribed. This is formally reflected by the fact that w3 does not belong to all elements of the neighborhood sphere N+(w1).

The next step is to prove the soundness of S0, namely that every theorem of S0 is valid in Km. We use an induction on the length of derivations in S0; first we show that all axiom-schemata of S0 are valid in Km and then that all rules of S0 preserve validity in Km. In some cases we can simply rely on well-known properties of standard neighborhood models, illustrated for instance in Chellas 1980.

**Theorem 1** For every ϕ ∈ L, if ⊨ S0 ϕ then Km ⊨ ϕ.

**Proof** Let ℳ = ⟨W, Wc, N+, N−, V⟩ be an arbitrary model in Km and w an arbitrary world (situation) in W. By definition of truth-conditions, ℳ, w ⊨ ϕ for any propositional tautology ϕ. This is enough for axiom (1). Suppose ℳ, w ⊨ cap(a1) for some a1 ∈ ACT; then, w ∈ V(cap(a1)) and, by P5, we have w ∈ V(amp(a1)), so ℳ, w ⊨ amp(a1), which is enough for axiom (3). Suppose ℳ, w ⊨ fixed(s1) ∧ sub(a2, s1), for some s1 ∈ SAC and a2 ∈ ACT; then |ϕ → und(s1)||ℳ ∈ N+(ϕ) and w ∈ V(sub(a2, s1)). By P6, |ϕ → (und(s1) → amp(a2))||ℳ ∈ N+(ϕ). This is enough for (4). One can similarly rely on P7 and P8 to show that every instance of (5) and (6) is true at w in ℳ. Moreover, properties P1–P4 guarantee the validity of (8), (9), (11) and (12), exactly as in standard neighborhood models. Finally, rules (2), (7) and (10) preserve validity (rule (2) also preserves truth at a world) in every neighborhood model.

Q.E.D.

Theorem 1, together with the fact that Km is a non-empty class of models (e.g., it includes the model for the citra sacrifice used above) guarantees that S0 is a consistent system.

The final step of our semantic characterization of S0 consists in proving that this system is complete w.r.t. the class Km, namely that every formula valid in Km is also a theorem of S0. We rely on the method of canonical models. We start by introducing some relevant terminology; what is here omitted for the sake of brevity can be found in Chellas 1980. So far we have been using a semantical notion of consistency for systems (a system X over L is consistent iff there exists a model for S); now, instead, we need to consider a syntactical notion of consistency: a system X over L is consistent iff there is at least some formula ϕ ∈ L s.t. ⊢ S0 ϕ (i.e., it is not the case that ⊬ S0 ϕ). It can be shown that the two notions of consistency (semantic and syntactical) are equivalent. Furthermore, we need to define notions of relative consistency. A set of formulas Γ ⊂ L is consistent with S0 iff there is no finite Γ′ ⊆ Γ s.t., if Γ′ = {ϕ1, ..., ϕn}, then ⊬ S0 ϕ1 ∧ ... ∧ ϕn. Finally, a set of formulas Γ ⊂ L is maximally consistent with S0 iff (I) Γ is consistent with S0 and (II) there is no Γ′ ⊂ L s.t. Γ ⊂ Γ′ and Γ′ is consistent with S0.

Let the model ℳS0 = ⟨W1, W2, N1, N2, V1⟩ be defined as follows:

\[24\] The caru is a cereal preparation offered in rituals. More details in Mylius 1995, s.v. and in Einoop 1985.
This property means that the relation of subsidiarity among an action \( ∅ = \text{all instances of axiom } (\phi) \) is satisfied due to the fact that every \( w ∈ W_{S_0} \) includes all instances of axiom (9). Indeed, suppose \( X ∈ N^+(w) \) for some \( X ≤ φ(W) \), then there is \( Y ∈ N(w) \) s.t. \( Y ⊆ X \) and \( Y = |φ|_{S_0} \) for some \( φ ∈ \mathcal{L} \). This means that \( X ≤ Y \). According to the usual properties of canonical models, we know that \( Y = |φ|_{S_0} = |φ|_{\mathfrak{M}_{S_0}} \), so \( \mathfrak{M}_{S_0}, w ∴ φ \) and, by axiom (9), \( \mathfrak{M}_{S_0}, w ∴ φ^\sim \); this entails \( |φ|_{S_0} = |φ|_0 \nvdash \mathfrak{M}_{S_0}, w ∈ N^+(w) \). If it were the case that \( X ∈ N^+(w) \), then, by P1, it would also be the case that \( Y ∈ N^+(w) \) and this would represent a contradiction. The property P4 is satisfied due to the fact that every \( w ∈ W_{S_0} \) includes all instances of axiom (12). Indeed, if it were the case that \( ∅ ∈ N^+(w) \), then, since for any \( φ ∈ \mathcal{L} \), \( ∅ = |φ|_{S_0} \), we would have \( |φ|_{S_0} = |φ|_{\mathfrak{M}_{S_0}} \) for some \( φ ∈ \mathcal{L} \), whence \( \mathfrak{M}_{S_0}, w ∴ φ \) and, which is impossible, given the validity of axiom (12). In the case of P5, let \( w ∈ V_{S_0}(\text{cap}(a_i)) \) for some \( a_i ∈ ACT \); hence \( \mathfrak{M}_{S_0}, w ∴ \text{cap}(a_i) \) and, by (3), we get \( \mathfrak{M}_{S_0}, w ∴ \text{amp}(a_i) \), which entails \( w ∈ V_{S_0}(\text{amp}(a_i)) \). In the case of P6, let \( w ∈ V_{S_0}(\text{sub}(a_j, s_i)) \) for some \( a_j ∈ ACT \) and \( s_i ∈ SAC \); furthermore, let \( |e \rightarrow \text{und}(s_i)|_{\mathfrak{M}_{S_0}} \) be a model used earlier for the \( \text{citrā} \) sacrifice, instead, is a standard one.

Taken together, Theorem 1 and Theorem 2 guarantee that \( S_0 \) is characterized by the class of models \( K_m \).

Models for system \( S_0 \) can be divided into two classes depending on whether they satisfy the following additional property:

P9 for any \( a_j ∈ ACT \) and \( s_i ∈ SAC \), either \( V(\text{sub}(a_j, s_i)) = W \) or \( V(\text{sub}(a_j, s_i)) = ∅ \).

This property means that the relation of subsidiarity among an action \( a_j \) and a sacrifice \( s_i \) either holds in all worlds/situations or in none. Models in which P9 is satisfied can be called standard models for \( S_0 \), those in which it is not satisfied non-standard models for \( S_0 \). The canonical model for \( S_0 \) used in Theorem 2 is actually a non-standard one, since there are at least two sets of formulas \( w \) and \( v \) which are maximally consistent with \( S_0 \) and s.t., for some \( a_j ∈ ACT \) and \( s_i ∈ SAC \), \( \text{sub}(a_j, s_i) \) w ∈ \( W \) while \( \text{sub}(a_j, s_i) \notin \mathfrak{M}_{S_0} \).

Notice that the model for the \( \text{citrā} \) sacrifice also validates the schema \( Oφ → Rφ \), i.e. (13), since for all situations \( v ∈ W \), we have \( N^+(v) = N^-(v) \). Thus, that model can be also used to interpret the system \( S_1 \) described in section 3.1, namely the system obtained by adding (13) to the axiomatic basis of \( S_0 \). This means that \( S_1 \) is a consistent system. In order to show the consistency of \( S_2 \), the other extension of \( S_0 \) mentioned in section 3.1 and obtained by adding the schema \( Oφ → ¬Rφ \), i.e. (14), to the axiomatic basis of \( S_0 \), we provide a model based on the argument illustrating the duty to perform the \( \text{agnihoṭra} \) sacrifice.

Anyone who has studied the Vedas, is physically and economically able to perform a sacrifice (i.e., meets the eligibility conditions) has to perform the \( \text{agnihoṭra} \) sacrifice. Since it is a fixed sacrifice, its subsidiaries need to be performed as much as possible. Among the subsidiary actions to be performed there is an offering of milk obtained from a cow which has a male calf. Let \( s_1 \) stand for the \( \text{agnihoṭra} \) sacrifice and \( a_1 \) for the offering of the milk. Here we do not need to take into account events or desired outcomes, so we can set \( EVT = OUT = ∅ \). Take a model \( \mathfrak{M} = (W, W_1, N^+, N^-, V) \) where \( W = \{w_1, w_2, w_3\}, W_1 = \{w_2, w_3\}, N^+(w_1) = \{W\}, N^-(w_1) = N^+(w_2) = N^-(w_2) = \{w_3\} = N^-(w_3) = ∅, V(\text{cap}(a_1, s_1)) = W, V(\text{amp}(a_1)) = \{w_2, w_3\} \) and \( V(\text{cap}(a_1)) = \{w_1\} \). First, we show that this model belongs to the class \( K_m \), so that it is a model for \( S_0 \). The validity of the properties P1-P4 is straightforward, given the definition of \( N^+ \) and \( N^- \). The validity of P5 is also easy to check, since \( a_1 \) is the only ritual action mentioned and \( V(\text{cap}(a_1)) ⊆ V(\text{amp}(a_1)) \). More precisely, \( \text{cap}(a_1) \) holds
only in the situation \(w_2\), whereas \(amp(a_1)\) holds in both \(w_2\) and \(w_3\) (for instance, because in \(w_3\) the milk is obtained from a cow which has only a female calf). The validity of \(P7\) and \(P8\) is trivial since \(\text{EVT} = \text{OUT} = \emptyset\). Thus, it only remains to check the validity of \(P6\): given that \(|\text{e} \rightarrow (\text{und}(s_1) \rightarrow \text{amp}(a_1))|^{\overline{\text{O}}} = |\text{e} \rightarrow (\text{und}(s_1) \rightarrow \text{amp}(a_1))|^{\overline{\text{R}}} = W\), then \(|\text{e} \rightarrow \text{und}(s_1)|^{\overline{\text{R}}} = |\text{e} \rightarrow (\text{und}(s_1) \rightarrow \text{amp}(a_1))|^{\overline{\text{R}}} \subseteq N^+(w)\) and this is enough. Finally, we have to show that the schema (14), i.e. \(\text{O} \phi \rightarrow \neg \text{R} \phi\), is valid in our model. The only relevant situation to consider is \(w_1\) (given that \(N^+(w_2) = N^-(w_2) = N^+(w_3) = N^-(w_3) = \emptyset\) and the result holds since \(N^+(w_1) \cap N^-(w_1) = \emptyset\). Thus, this is a model for \(S_2\) and the latter is a consistent system, as claimed in section 3.1. Notice that in the model for the \textit{agnihotra} sacrifice all strong and weak Vedic duties considered are observed in the three situations \(w_1\), \(w_2\) and \(w_3\); indeed, the three situations belong to every element of their neighborhood spheres (in most cases, trivially, since all neighborhood spheres except \(N^+(w_1)\) are empty).

3.3 Variation: dyadic deontic operators

Vedic injunctions concerning the performance of ritual actions can be formalized also in terms of \textit{dyadic deontic operators}, namely in a language in which \(\text{O}\) and \(\text{R}\) always take two formulas as arguments rather than one. Formulas whose main operator is either dyadic \(\text{O}\) or dyadic \(\text{R}\) will be here represented as \(\text{O} \phi \phi\) and \(\text{R} \phi \phi\); another common notation is \(\text{O} (\phi / \theta)\) and \(\text{R} (\phi / \theta)\). Notice that in both cases \(\phi\) represents the main argument and \(\theta\) the antecedent or \textit{triggering condition} (despite being positioned after \(\phi\) in the second notation; that is due to alternative natural language translations of these formulas, such as ‘it should be the case that condition \(\theta\) leads one to bring about \(\phi\)’ versus ‘it should be the case that one brings about \(\phi\) under condition \(\theta\)’).

In the present context, dyadic operators can be useful to express the fact that eligibility is always a triggering condition for the performance of a sacrifice. Moreover, according to Mimamsa authors, each Vedic injunction addresses a person insofar as he or she is identified by a particular desire. In this perspective, desire is an essential element of the ‘eligibility’ (\textit{adhipakara}, see on this point \textit{Freschi 2007}) so much that, when no desire is explicitly mentioned in connection with an injunction, these authors explain that a desire for happiness can be assumed to fill this necessary slot (such procedure is called \textit{visvajnayya} and is discussed in \textit{Ezechic Aphorisms 4.3.15–16}). A similar argument actually leads to two possible solutions to formalize the Mimamsa theory. One solution consists in taking the desire for happiness as redundant, following the idea that this desire is shared by all human beings. The other solution consists in making explicit reference to the desire for happiness when no other specific desire is mentioned. In section 3.1 we opted for the first solution; here, for a comparison, we will opt for the second solution and show how the definition of fixed and occasional sacrifices can be changed by adding ‘happiness’ as a special desired outcome. Before doing that, however, we would like to mention another issue concerning the move from monadic to dyadic deontic operators: the treatment of deontic dilemmas.

Dyadic deontic operators are exploited in \textit{Ciabattoni et al. 2015} to formalize an argument discussed by Mimamsa authors, which can be called the \textit{siyena dilemma}. Such dilemma pertains to the performance of the \textit{siyena} sacrifice, a malefic sacrifice meant for harming one’s enemy. Mimamsa authors all agree that it should not be performed because it violates the prohibition to harm living beings. \textit{Ciabattoni et al. 2015} argue that the seeming conflict between the prohibition to harm any living being and the injunction to perform the \textit{siyena} under the desire to harm an enemy can be formally solved through a dyadic operator. Indeed, it is possible to build a formal system, like \textit{bMDL}, in which the dilemma is represented by a conjunction of formulas \(\text{O} \top \neg \phi \land \text{O} \psi \phi\), where \(\psi\) can be read ‘you harm some living being’ (which is a necessary consequence of the performance of the \textit{siyena}), \(\psi\) ‘you desire to harm your enemy’ and \(\top\) represents any tautology. The point is that from the conjunction \(\text{O} \neg \phi \land \text{O} \psi \phi\) one cannot infer, in \textit{bMDL}, both \(\text{O} \neg \phi\) and \(\text{O} \psi \phi\), which would lead to the conclusion that under the same condition \(\psi\) (‘you desire to harm your enemy’) you should both harm and not harm other living beings.

Within the conceptual framework developed in this article, which allows one to distinguish between \textit{strong} and \textit{weak} Vedic duties, we can provide an alternative analysis of the \textit{siyena} dilemma. Indeed, since the \textit{siyena} sacrifice has to be performed when a desire for something different from happiness arises (harming an enemy), then it can be classified as an elective sacrifice. This means, according to our analysis in section 2.1.4, that the injunction to perform it expresses a \textit{weak} duty or recommendation. On the other hand, the injunction to avoid harming any living being represents a generic prohibition. Thus, in the present conceptual framework the dilemma would no longer involve two obligations but a

\[\text{For an introduction to the use of dyadic operators in deontic logic, see Hilpinen and McNamara 2013. We adhere to the notation in Åqvist 1987.}\]
recommendation and a prohibition: one could say that the šgēna sacrifice is something whose performance is recommended in situations that are not deontically ideal, since they are situations in which one desires something that is, in general, prohibited (harming some living being and, more specifically, an enemy).²⁶

The possible applications of dyadic operators discussed so far, including the treatment of deontic dilemmas, suggest a generalization of the formal apparatus of section 3.1. We briefly illustrate how this generalization can be achieved. Let $\mathcal{L}d$ be the language obtained by replacing monadic $O$ and $R$ in $\mathcal{L}$ with their dyadic analogue. The set of wffs of $\mathcal{L}d$ is then the smallest set satisfying the following properties:

- $\delta \alpha$ is a wff, for any $\delta \alpha \in ART$;
- if $\phi$ is a wff, then so is $\neg \phi$;
- if $\phi$ and $\psi$ are wffs, then so are $(\phi \rightarrow \psi)$, $O_\psi(\phi)$ and $R_\psi(\phi)$.

Parentheses can be eliminated when there is no risk of ambiguity, as usual. The meaning of $O_\psi(\phi)$ and $R_\psi(\phi)$ is, respectively, ‘in all situations in which strong Vedic duties are observed, $\phi$ is the case under condition $\psi$’ and ‘in all situations in which weak Vedic duties are observed, $\phi$ is the case under condition $\psi$’. Our simplified reading, then, turns out to be: ‘according to the Vedas, $\phi$ is obligatory given condition $\psi$’ and ‘according to the Vedas, $\phi$ is recommended given condition $\psi$’.

Since here we want to formalize injunctions by making explicit reference to the desire for happiness when no other specific desire is mentioned, then we can assume that the set of outcomes OUT (which is inherited, by definition of $\mathcal{L}d$, from the language $\mathcal{L}$ of section 3.1) includes a special individual constant $o^*$ denoting happiness and we can provide the following definitions of fixed, occasional and elective sacrifices:

- $\text{fixed}(s_i) \overset{\text{def}}{=} O_{\xi \land \text{des}(\phi^*)}(\text{und}(s_i))$;
- $\text{occasional}(s_i)/e_n \overset{\text{def}}{=} O_{\xi \land \text{tpl}(e_n) \land \text{des}(\phi^*)}(\text{und}(s_i))$;
- $\text{elective}(s_i)/o_n \overset{\text{def}}{=} R_{\xi \land \text{des}(o_n)}(\text{und}(s_i))$.

The first definition says that $s_i$ denotes a fixed sacrifice iff in all situations in which strong Vedic duties are observed, eligibility is met and happiness is desired, $s_i$ is undertaken. The second definition says that $s_i$ denotes an occasional sacrifice iff in all situations in which strong Vedic duties are observed, eligibility is met, the relevant occasion takes place and happiness is desired, $s_i$ is undertaken. The third definition says that $s_i$ denotes an elective sacrifice iff in all situations in which weak Vedic duties are observed, eligibility is met and the relevant outcome is desired, $s_i$ is undertaken. These definitions differ from the original ones in section 3.1 due to the presence of $o^*$ (happiness) when no other outcome is desired and to the fact that the dyadic notation replaces the main material implication in the scope of $O$ and $R$.

We introduce now a formal system called $S_0d$, which is the dyadic analogue of $S_0$. We can borrow the principles (1), (2) and (3) directly from $S_0$. The principles (4), (5) and (6), instead, need to be modified by replacing the main material implication in the scope of $O$ with the dyadic notation and by taking into account $o^*$ when fixed and occasional sacrifices are mentioned. For the sake of simplicity, we will refer to the modified principles with the same numbers used for the original ones. For instance, the schema (5) becomes: $(\text{occasional}(s_i)/e_n \land \text{sub}(a_j, s_i) \rightarrow O_{\xi \land \text{tpl}(e_n) \land \text{des}(\phi^*)}(\text{und}(s_i)) \rightarrow \text{amp}(a_j))$.

In order to axiomatize the properties of the dyadic operator $O$, we choose to rely again on the preliminary system $bMDL$ developed in Ciabattoni et al. 2015. In the Appendix to this article it is shown that we can extract from $bMDL$ the dyadic versions of the principles (7), (8) and (9),²⁷ together with the following rule:

$$\theta \equiv \xi \quad O_\psi \phi \equiv O_\xi \phi$$

Similarly, as far as the principles for dyadic $R$ are concerned, we take the dyadic versions of the principles (10), (11) and (12),²⁸ together with the rule below:

$$\theta \equiv \xi \quad R_\psi \phi \equiv R_\xi \phi$$

The axiomatic basis of $S_0d$ is finally defined by the list of (modified) principles (1)-(12), together with (15) and (16).

²⁶ Prohibitions cannot be simply defined as negative obligations in the Mīmāṃsā theory, since prohibitions are defined insofar as their transgression leads to sanctions, whereas obligations, even negative obligations, lead to a result. For a systematic overview of deontic commands in Mīmāṃsā, see Freschi and Pascucci 2019.

²⁷ Namely, $O_\psi \equiv O_\xi \quad O_\psi \phi \equiv O_\xi \phi \quad O_\phi \psi \rightarrow (O_\phi \phi \land O_\phi \psi)$ and $\neg (O_\phi \phi \land O_\phi \neg \phi)$; we will keep the original numbers for reference.

²⁸ Namely, $R_\psi \equiv R_\xi \quad R_\psi \phi \equiv R_\xi \phi \quad R_\phi \psi \rightarrow (R_\phi \phi \land R_\phi \psi)$ and $\neg R_\phi \phi \land R_\phi \neg \phi)$; we will keep the original numbers for reference.
In the rest of this section we sketch how the semantic characterization of \( S_0 \) can be modified in order to get a semantic characterization of \( S_{\mathcal{O}} \). Models are now to be defined as structures of kind \( \mathfrak{M} = (W, W_\tau, \{ N_\theta^+: \theta \in Ld \}, \{ N_\theta^-: \theta \in Ld \}, V) \). Thus, for every situation \( w \in W \) and every formula \( \theta \in Ld \) we have now a pair of neighborhood spheres, \( N_\theta^+(w) \) and \( N_\theta^-(w) \). Truth conditions are as in models for \( S_0 \), except those concerning formulas whose main operator is dyadic \( \mathcal{O} \) or dyadic \( \mathcal{R} \):

\[-2M, w \models \mathcal{O}_\phi \text{ iff } ||\phi||^{2M} = \{ v \in W : 2M, v \models \phi \} \in N_\theta^+(w); \]
\[-2M, w \models \mathcal{R}_\phi \text{ iff } ||\phi||^{2M} = \{ v \in W : 2M, v \models \phi \} \in N_\theta^-(w). \]

Let \( K_{m,d} \) be the class of models satisfying a modified version of the properties P1-P8 of models in \( K_{m} \) (see section 3.2), obtained in the following way:

- in P1 and P3 replace any occurrence of \( N^+(w) \) with \( N^+_\theta(w) \), for an arbitrary \( \theta \in Ld; \)
- in P2 and P4 replace any occurrence of \( N^-(w) \) with \( N^-\theta(w) \), for an arbitrary \( \theta \in Ld; \)
- in P6 replace \( [\varepsilon \to \text{und}(s_i)] ||^{2M} \in N^+(w) \) with \( ||[\text{und}(s_i)]||^{2M} \in N^+_{\varepsilon,\text{des}(\tau)}(w) \) and \( ||[\varepsilon \to (\text{und}(s_i) \to \text{amp}(a_j))]||^{2M} \in N^+_{\varepsilon,\text{des}(\tau)}(w) \);  
- in P7 replace \( ||(\varepsilon \& \text{tp}(e_n)) \to \text{und}(s_i)]||^{2M} \in N^+(w) \) with \( ||[\text{und}(s_i)]||^{2M} \in N^+_{\varepsilon,\text{tp}(e_n) \& \text{des}(\tau)}(w) \) and \( ||(\varepsilon \& \text{tp}(e_n)) \to (\text{und}(s_i) \to \text{amp}(a_j))||^{2M} \in N^+(w) \) with \( ||[\text{und}(s_i) \to \text{amp}(a_j)]||^{2M} \in N^+_{\varepsilon,\text{tp}(e_n) \& \text{des}(\tau)}(w) \);  
- in P8 replace \( ||(\varepsilon \& \text{des}(o_n)) \to \text{und}(s_i)]||^{2M} \in N^+(w) \) with \( ||[\text{und}(s_i)]||^{2M} \in N^+_{\varepsilon,\text{des}(o_n)}(w) \) and \( ||[\varepsilon \& \text{des}(o_n)) \to (\text{und}(s_i) \to \text{cap}(a_j))||^{2M} \in N^+(w) \) with \( ||[\text{und}(s_i) \to \text{cap}(a_j)]||^{2M} \in N^+_{\varepsilon,\text{des}(o_n)}(w) \).

Furthermore, let models in \( K_{m,d} \) satisfy the following additional property:

**P10** if \( \vdash_{S_{\mathcal{O}}} \theta \equiv \xi \), then \( N_{\theta}^+(w) = N_{\xi}^+(w) \) and \( N_{\theta}^-(w) = N_{\xi}^-(w) \).

It can be easily shown that there is at least one model in the class \( K_{m,d} \). We provide here a model based on the argument used to enjoin the performance of the rājaṇiṣṭi sacrifice. A person who desires to improve his sight and fulfills the other relevant criteria for eligibility (e.g., he or she has studied the Vedas, is physically able and is wealthy enough) is the addresser of the injunction to perform the rājaṇiṣṭi. Since this is an elective sacrifice, one can reach its result only by performing all its auxiliaries exactly; in this case, the offering of two eight-cup cakes for Agni bhrājavan and the offering of a caru for Śūrya. Let \( s_1 \) stand for the rājaṇiṣṭi sacrifice, \( a_1 \) for the offering of two eight-cup cakes for Agni bhrājavan, \( a_2 \) for the offering of a caru for Śūrya and \( a_1 \) for the outcome of improving one’s sight. Here we do not need to take into account events so we can set \( \text{EV}_T = \emptyset \). Take a model \( \mathfrak{M} = (W, W_\tau, \{ N_\theta^+: \theta \in Ld \}, \{ N_\theta^-: \theta \in Ld \}, V) \) where \( W = \{ w_1, w_2, w_3 \} \), \( W_\tau = W \), \( N^+_{\varepsilon,\text{des}(o_1)}(w_1) = \{ w_1, w_2, w_3 \}, \mathcal{N}^-_{\varepsilon,\text{des}(o_1)}(w_1) = \{ w_1, w_2, w_3 \}, N^+_{\varepsilon,\text{des}(o_1)}(w_2) = N^+_{\varepsilon,\text{des}(o_1)}(w_3) = N^-_{\varepsilon,\text{des}(o_1)}(w_2) = N^-_{\varepsilon,\text{des}(o_1)}(w_3) = \emptyset \forall \xi \in Ld \) s.t. \( \xi \neq (\varepsilon \& \text{des}(o_1)) \), \( N_{\varepsilon}^+(w_2) = N_{\varepsilon}^+(w_3) = N_{\varepsilon}^-(w_2) = N_{\varepsilon}^-(w_3) = \emptyset \forall \theta \in Ld \) and \( \text{V}(W_{\text{sa}}(a_1, s_1)) = \text{V}(w_1), \text{V}(\text{und}(s_1)) = \text{V}(\text{amp}(a_1)) = \text{V}(w_1) \). For every \( \varepsilon \in Ld \), \( P_5 \) holds as well, since \( a_2 \) and \( a_1 \) are the only two ritual actions mentioned and \( \text{V}(\varepsilon, \text{cap}(a_1)) = \text{V}(\text{amp}(a_1)) \). More precisely, \( \text{cap}(a_1) \) only holds in the situation \( w_1 \), whereas \( \text{amp}(a_1) \) holds in the situations \( w_1 \) and \( w_2 \) (for instance, because in \( w_1 \) one offers two five-cup cakes, rather than two eight-cup cakes, for Agni bhrājavan). The validity of \( P_6 \) and \( P_7 \) is trivial since \( N^+_{\varepsilon,\text{des}(\tau)}(w) = \emptyset \forall w \in W \) and \( \text{EV}_T = \emptyset \). It only remains to show the validity of \( P_8 \) and the only situation to check is \( w_1 \): given that \( ||\text{und}(s_1)||^{2M} = \{ w_1, w_2 \} \), \( ||\text{und}(s_1) \to \text{cap}(a_1)||^{2M} = \{ w_1, w_2 \} \) and \( ||\text{und}(s_1) \to \text{amp}(a_1)||^{2M} = \{ w_1, w_3 \} \) and \( ||\text{und}(s_1) \to \text{amp}(a_1)||^{2M} = ||\text{und}(s_1) \to \text{cap}(a_1)||^{2M} = \{ w_1, w_2 \} \) for every \( \theta \in Ld, ||\text{und}(s_1)||^{2M} \in N^+_{\varepsilon,\text{des}(o_1)}(w_1) \) and this is enough. Notice that in the model for the rājaṇiṣṭi sacrifice \( w_1 \) is a situation in which all strong and weak Vedic duties mentioned are observed; indeed, it belongs to all elements of all its neighborhood spheres. The same applies, trivially, also to \( w_2 \) and \( w_3 \).

The soundness of \( S_{\mathcal{O}} \) w.r.t. the class \( K_{m,d} \) can be easily checked; the only relevant remark is that P10 guarantees that (15) and (16) preserve validity in all models of \( K_{m,d} \). As far as completeness is concerned, a canonical model \( \mathfrak{M}_{S_{\mathcal{O}}} \) for \( S_{\mathcal{O}} \) can be defined in analogy with the model \( \mathfrak{M}_{S_0} \) of section 3.2, except for the properties concerning neighborhood spheres:

- for every \( w \in W_{S_{\mathcal{O}}}, N_{\theta}^+(w) = \{ X \subseteq \phi(W) : \text{for some } \phi \in Ld, ||\phi||_{S_{\mathcal{O}}} \subseteq X \text{ and } \mathcal{O}_\phi \phi \in w \}; \)
- for every \( w \in W_{S_{\mathcal{O}}}, N_{\theta}^-(w) = \{ X \subseteq \phi(W) : \text{for some } \phi \in Ld, ||\phi||_{S_{\mathcal{O}}} \subseteq X \text{ and } \mathcal{R}_\phi \phi \in w \}. \)

It can be finally verified that \( \mathfrak{M}_{S_{\mathcal{O}}} \) satisfies the modified version of P1-P8 and P10, so that it belongs to the class \( K_{m,d} \). Here we just consider the case of P10. Assume that \( \vdash_{S_{\mathcal{O}}} \theta \equiv \xi \); then, by properties of
canonical models, for every world \( w \in W_{S_{0d}} \), we have \( \mathfrak{M}_{S_{0d}}, w \models \theta \equiv \xi \); from this, by (15) and (16) one gets that for every formula \( \phi \in \mathcal{L}_{d} \), \( \mathfrak{M}_{S_{0d}}, w \models (O_\theta \equiv O_\xi \phi) \land (R_\theta \phi \equiv R_\xi \phi) \). Thus, (I) \( ||\phi||_{\mathfrak{M}_{S_{0d}}} \in N_\theta^+ \) iff \( ||\phi||_{\mathfrak{M}_{S_{0d}}} \in N_\xi^+ \) and (II) \( ||\phi||_{\mathfrak{M}_{S_{0d}}} \in N_\theta^- \) iff \( ||\phi||_{\mathfrak{M}_{S_{0d}}} \in N_\xi^- \). From this we get the intended result, namely that (I) \( N_\theta^+(w) = N_\xi^+(w) \) and (II) \( N_\theta^-(w) = N_\xi^-(w) \).

We conclude this section with a remark on useful variations of \( S_{0d} \). We argued that one of the reasons to move from monadic deontic operators to dyadic ones is the possibility of treating deontic dilemmas. For instance, as illustrated in the analysis of the \( \text{śyena} \) dilemma in Ciabattioni et al., 2015, one can block problematic inferences from a generic triggering condition to a more specific one; however, if the only principles relating triggering conditions are those formalized by (15) and (16), there is some risk that too many inferences are blocked. For instance, it seems plausible to assume that sometimes the addition of a triggering condition preserves the injunction to perform an action. Consider the case of the \( rājaniśṭi \) sacrifice discussed above. If it is the case that a person should perform the \( rājaniśṭi \) when he or she is eligible and desires to improve his or her sight, then it seems also plausible that that person should perform the \( rājaniśṭi \) when, in addition to the two conditions mentioned, he or she also has a generic desire for happiness. However, in the system \( S_{0d} \) over the language \( \mathcal{L}_{d} \) it is not possible to make this inference, since the formula \( R_{\theta \land \text{des}(s_1)}(\text{und}(s_1)) \) does not entail the formula \( R_{\theta \land \text{des}(s_1) \land \text{des}(o_1)}(\text{und}(s_1)) \), where \( s_1 \) and \( o_1 \) are defined as in the \( rājaniśṭi \) example and \( o^* \) denotes happiness. Similar issues arise with dyadic \( O \) (already in the system \( \text{bMDL} \), whence we extracted our principles for this operator). Thus, there is some need to add further principles relating triggering conditions in systems built over \( \mathcal{L}_{d} \). Let \( S_{0d^*} \) be the system obtained by extending the axiomatic basis of \( S_{0d} \) with the two principles below:

\[
O_{\psi} \phi \to O_{\psi \land \text{des}(o^*)} \phi
\]

(17)

\[
R_{\psi} \phi \to R_{\psi \land \text{des}(o^*)} \phi
\]

(18)

The meaning of these two principles is intuitive: if it is a (strong or weak) Vedic duty to perform \( \phi \) under condition \( \psi \), then it is also a (strong or weak) Vedic duty to perform \( \phi \) under the condition \( \psi \) and the desire for happiness. These two principles guarantee, at least, that strong and weak duties are always preserved under the addition of the desire for happiness. On the other hand, if one wants to claim that the desire for happiness not only preserves duties, but makes no difference at all with respect to duties, then she can add to the axiomatic basis of \( S_{0d^*} \) also the following principles:

\[
O_{\phi \land \text{des}(o^*)} \phi \to O_{\phi} \phi
\]

(19)

\[
R_{\phi \land \text{des}(o^*)} \phi \to R_{\phi} \phi
\]

(20)

The result is a system that we can call \( S_{0d^{**}} \). This system brings one back to the idea that reference to the desire for happiness is redundant and provides also a formal justification of this position. Indeed, even if \( o^* \) occurs in the definition of fixed and occasional sacrifices, in \( S_{0d^{**}} \) any formula in which \( o^* \) occurs as a triggering condition (having the shape \( O_{\phi \land \text{des}(o^*)} \phi \) or \( R_{\phi \land \text{des}(o^*)} \phi \)) is logically equivalent to the formula obtained by removing \( o^* \) from the set of triggering conditions (having the shape \( O_{\phi} \phi \) or \( R_{\phi} \phi \)).

4 Maṇḍana Miśra’s interpretation of Vedic duties

According to our analysis in section 2, the deontic theory of Vedic sacrifices developed in Common Mīmāṃsā is characterized by the following features:

1. it seems to entail several kinds of Vedic duties, which can be classified according to the fundamental distinction between strong and weak duties;
2. it seems to entail three irreducible classes of ritual actions, namely fixed, occasional and elective ones;
3. it seems to entail a role for both results and sanctions, though without specifying it clearly.

We have seen how these distinctions are logically connected, that is, the principles that, according to Mīmāṃsā, allow us to infer certain features of an action from, for example, the fact that it is a subsidiary of an elective sacrifice. We have not, however, seen any attempt from within Common Mīmāṃsā to ‘rationalize’ this system of principles, by which we mean both giving a rational motivation for the principles themselves (e.g., what is the reason why subsidiaries of fixed sacrifices only need to be performed to the best of one’s ability, in contrast to the subsidiaries of elective sacrifices, which need to be performed exactly as the texts prescribe?) as well as identifying and eliciting the basic concepts on which the system is based.
The eighth-century Mīmāṃsā author Maṇḍana Miśra offered just such a rationalization in his *Analysis of Injunctions* (Vidhiviveka). He argued that what we called the Common Mīmāṃsā system could be streamlined by interpreting the meaning of injunctions (vidhitīṁpratyāyārtha) in a uniform way across all cases. The reduction that he effected has a twofold sense:

- the reduction of multiple concepts of injunction, and classes of actions that can be enjoined, to a single concept of injunction and a homogeneous class of actions. This deals with the issues enunciated above as point 1 and point 2;
- the reduction of deontic concepts to non-deontic concepts, and in particular, the reduction of the meaning of injunctions to a form of instrumentality, namely to the idea that enjoined actions are instruments to get a desired result. In this way, he attempted to rethink the role of results and addressed the issue raised above as point 3.

In other words, Maṇḍana reduced the various types of sacrifices to a single case, insofar as he argued that all sacrifices are based on a desire. Furthermore, he reinterpreted injunctions in non-deontic terms as just stating that the prescribed action is an instrument to realise a desired output. In this way, Maṇḍana replaced multiple notions of duty and injunctions with one by reducing the notion of duty to that of instrumentality towards a desired result, and the notion of injunction to that of a statement that enunciates an action’s instrumental relation to a desired result.

The essential point for Maṇḍana is that all three classes of action share the property of leading to a certain result. We can articulate his reduction in four steps:

*Step one: All sacrifices have a result.* Maṇḍana’s intervention is that fixed sacrifices lead to fixed results (i.e., stable, permanent results, such as happiness), and elective sacrifices lead to non-fixed results (i.e., things that happen once, such as the acquisition of cattle).

*Step two: Occasional sacrifices are deontically identical to fixed sacrifices.* This is implicit already in early Mīmāṃsā, so Maṇḍana did not have to argue at length for this position. The basic idea is that both fixed and occasional sacrifices have an occasion. The occasion for the former recurs on a regular basis, whereas the occasion for the latter does not.

One does not need to be concerned about the fact that, for example, the occasion of the *agnihotra* sacrifice takes effect every day, while the occasion of the *jyotiṣṭoma* sacrifice takes effect every year and the occasion of the *jāteṣṭi* sacrifice takes effect only a couple of times in one’s life. This position is supported by the following observation:

An occasion is that of which the presence makes the performance of an action necessary. How is it so? The principal cause (in this case, the occasion) is the one which, being present, [the result] arises, all other [causes] are secondary (similarly, when the occasion is present, the sacrifice needs to be performed). An occasion is not brought about by the agent’s effort (anupādeya), in the sense that its occurrence is completely independent of the agent’s will and desire. Maṇḍana considered ‘living’ (jīvāna) the occasion of both the fixed and occasional rituals. In both cases ‘living’ is qualified by a particular time. The difference is that the time of the fixed rituals is simply daily (i.e., every evening, every morning, etc.), whereas the time of the so-called occasional rituals is either less frequent (e.g., every springtime) or less predictable (e.g., upon the birth of a son).

An action whose necessary performance is ‘triggered’ by an occasion does not need to be performed with all of its subsidiaries present. Rather, the action to be performed is to be provided with only those subsidiaries that it is feasible for the agent to include. Generally an injunction conveys that one who performs the enjoined act, when the occasion is present, will accomplish the end he desires. Maṇḍana maintained that he will always accomplish this end so long as he performs the enjoined action in some way or another (yathākathānacād). When the occasion is ‘living’, we know, additionally, that in order for the action to be accomplished on the given occasion (i.e., throughout one’s life), some allowances have to be made for times of adversity, hardship, etc., and therefore the ‘good enough’ principle is invoked (see below, step four).

---

30 *Analysis of Injunctions* p. 256: katham? yasmin satī bhavaty eva sa helur mukhyāḥ, itarasa tu bhaktiyo.
31 *Analysis of Injunctions* p. 245: jīvānāder nimittasya sāmyāt kālo viśeṣakaḥ | nimittārthas tatra jāte jāte karmmaṇo ‘vaśyakāryatā |.
32 *Analysis of Injunctions* p. 257: avajjakarttavyatā ca karmmaṇaḥ praitūtā nopaśāyanāhivaḥkanbhāvaśa maṅgale, na kadācid ahiḥtiṣyayet ato yathākātyaṅgasa naśaṁṣeta sarvābhilāsiyopāyaṁ karmeti gaṁgante.
33 *Analysis of Injunctions* p. 257: nimittatā ca karttavyatāyāḥ vādhīṛpaṁ, yasmin satī tatlurvaṁ yathākathānacād sadā samāhitatā enaṅkṣayaṁ śadhaṣati.


Step three: Fixed and occasional sacrifices have a result that is always desired. Maṇḍana argued that it is necessary in Mīmāṃsā to connect even the fixed and occasional sacrifices with a result, namely happiness. The result is also necessary to connect the principal act (e.g., the performance of the full- and new-moon sacrifices) with subsidiary acts (e.g., the performance of the preliminary sacrifices), since the latter assist in bringing about the result of the former, which therefore in their case fulfills the role of the desired end. When a result is not specifically mentioned it has to be postulated.

Step four: Elective sacrifices have a result that is sometimes desired. The result of these sacrifices is something that one happens to desire in non-ordinary circumstances, for instance rain in case of drought.

Thus, summing up, a result which one constantly desires motivates one to undertake a fixed sacrifice throughout one’s life and an occasional sacrifice whenever the relevant occasion occurs; a result which one contingently desires motivates one to undertake the corresponding elective sacrifice whenever one desires its result. Maṇḍana’s proposal seems to entail that all injunctions actually express recommendations: injunctions can be simply taken as recipes useful to reach a desired goal. Notice that, according to our analysis, recommendation is the kind of deontic property that can be ascribed to elective sacrifices and, in Maṇḍana’s perspective, all sacrifices, like the elective sacrifices of Common Mīmāṃsā, are performed only because one desires their result. Thus, Maṇḍana’s argument first eliminates any reference to obligations and then suggests an alternative reading for recommendations: something is recommended if it is an instrument to achieve a desired goal. It can be argued that his attempt to provide a completely uniform account of all sacrifices could undermine the distinction between subsidiary actions that have to be performed exactly as prescribed and subsidiary actions that have to be performed according to one’s possibility (the ‘good enough’ principle). Maṇḍana was very concerned, however, to retain the interpretive and ritual conclusions of Common Mīmāṃsā, while putting them on a new theoretical foundation. Therefore he was at pains to distinguish subsidiaries that have to be performed exactly as enjoined and those that can be performed as much as possible within his new framework, and to this end, he invoked the principle that the Veda cannot enjoin the performance of something that is physically impossible, and therefore allowances must be made for cases of exigency.

From a formal point of view, Maṇḍana’s argument could be taken into account by replacing the definitions of fixed and occasional sacrifices in section 3.1 with those below (where $o^*$, as usual, represents happiness):

- $\text{fixed}(s_i) = \text{def} \ R((e \land \text{des}(o^*)) \rightarrow \text{und}(s_i));$
- $\text{occasional}(s_i)/e_n = \text{def} \ R((e \land \text{tpl}(e_n) \land \text{des}(o^*)) \rightarrow \text{und}(s_i));$

The main differences with respect to the old definitions are:

- the use of $R$ rather than $O$ as the main operator in the definiens of fixed and occasional sacrifices;
- reference to the desire for happiness as a triggering condition in the definiens of fixed sacrifices and occasional sacrifices. \(^{34}\)

Alternatively, in a dyadic setting, one can replace the definitions of fixed and occasional sacrifices in section 3.2 (which already take into account the role of the desire for happiness) with the two below:

- $\text{fixed}(s_i) = \text{def} \ R_{e \land \text{des}(o^*)}(\text{und}(s_i));$
- $\text{occasional}(s_i)/e_n = \text{def} \ R_{e \land \text{tpl}(e_n) \land \text{des}(o^*)}(\text{und}(s_i)).$

Both solutions allow one to capture the idea that, according to Maṇḍana, Vedic injunctions to perform sacrifices have to be always taken as recipes rather than orders. Thus, they have the deontic force of a recommendation, rather than an obligation.

5 Conclusion

The article shows some of the results of applying conceptual analysis and formal logic to the understanding of a premodern philosophical tradition and how one gains through this exercise both clarity in one’s understanding of the tradition and new insights, in general, regarding logic and deontic reasoning. We saw how one can make sense of the Mīmāṃsā distinction between the duties to perform fixed, occasional and elective sacrifices by focusing on the interplay between certain deontic concepts, such as those of strong duty and weak duty (obligation and recommendation), the role of primary and subsidiary ritual actions, the eligibility criterion and the ‘good enough’ principle.

\(^{34}\) The idea of making explicit reference to a desire in each sacrifice has been followed already in the formalism of section 3.3. Maṇḍana strengthens the Common Mīmāṃsā principle that desires are always present as triggering conditions for the performance of sacrifices, by saying that one undertakes sacrifices only in order to fulfill a desire.
We proposed formalisms that can be used to represent arguments discussed by Mīmāṃsā authors in a rather easy way and introduced logical systems capturing different properties of weak duties and strong duties. These ideas can be further developed to reach a more refined analysis of the deontic theory outlined by Mīmāṃsā. Furthermore, as a side aspect, we illustrated some connection between the formal systems employed here and a system used in earlier works on Mīmāṃsā, bMDL (see also the Appendix below, which is dedicated to the proof of how one can extract axioms and rules for the operator $\mathcal{O}$ from the latter system).

Finally, the analysis elaborated in this article gave us a philosophical perspective onto the intervention of Maṇḍana on the Common Mīmāṃsā system, which turned out to be a simplification of the earlier description of ritual actions and a reduction of various deontic notions of duty to a non-deontic notion of instrumentality. Future research will aim at developing a detailed formal analysis of Maṇḍana’s intervention.

Appendix: properties of the operator $\mathcal{O}$

From Mīmāṃsā texts to axioms

In Ciabattoni et al. (2015), the authors introduce a system called bMDL (basic Mīmāṃsā deontic logic), which is obtained by adding to (any axiomatization of) the alethic system $\mathbf{S4}$ the following axioms, where $\Box$ is the alethic operator for necessity:

\begin{align}
\Box(\phi \rightarrow \psi) \land \mathcal{O}_\theta \phi & \rightarrow \mathcal{O}_\theta \psi 
\tag{21}
\\ 
\Box(\psi \rightarrow \neg \phi) \rightarrow \neg(\mathcal{O}_\theta \phi \land \mathcal{O}_\theta \psi) & 
\tag{22}
\\ 
(\Box((\psi \rightarrow \theta) \land (\theta \rightarrow \psi)) \land \mathcal{O}_\psi \phi) & \rightarrow \mathcal{O}_\theta \phi 
\tag{23}
\end{align}

The above axioms arise by formalizing some of the principles (called nyāyas) lied down by Mīmāṃsā authors to interpret the Vedas independently of any authorial intention. Nyāyas are metarules introduced implicitly or explicitly by Mīmāṃsā authors in order to bring order in their exegesis of Vedic sacrificial prescriptions and are in this sense a plausible starting point to introduce axioms for a logical system.

Axiom (21) arises from three different principles; among them there is the following abstraction of a nyāya found in Tantrarahasya IV.4.3.3 (about which see Freschi 2012):

If the accomplishment of X presupposes the accomplishment of Y, the obligation to perform X prescribes also Y.

Axiom (22) arises from the so-called principle of the half-hen, which is implemented in different Mīmāṃsā contexts (e.g., Kumārila’s subcommentary on Commentary on Exegetic Aphorisms 1.3.3); an abstract representation of it is:

Given that purposes Y and Z exclude each other, if one should use item X for the purpose Y, then it cannot be the case that one should use it at the same time for the purpose Z.

Finally, Axiom (23) arises from a discussion (in ŚBh on Exegetic Aphorisms 6.1.25) on the eligibility to perform sacrifices, which can be abstracted as follows:

If conditions X and Y are always equivalent, given the duty to perform Z under the condition X, the same duty applies under Y.

The notion of entailment involved in these principles is formalized in Ciabattoni et al. 2015 in terms of strict implication: the claims ‘$p$ presupposes $q$’, ‘$p$ and $q$ exclude each other’ and ‘$p$ and $q$ are always equivalent’ are respectively rendered as $\Box(p \rightarrow q), \Box(p \rightarrow \neg q)$ and $\Box(p \equiv q)$.

35 See Chellas 1980 for a presentation of $\mathbf{S4}$ and other alethic modal systems.

36 It is noteworthy that according to our analysis of Maṇḍana’s approach in section 4, it would be more convenient to reformulate axiom (22) as follows:

$$\Box(\theta \rightarrow \neg \xi) \rightarrow \neg(\mathcal{O}_\theta \phi \land \mathcal{O}_\xi \phi)$$

In this way, the antecedent would say something about triggering conditions rather than instruments (and the purposes mentioned in the principle of the half hen are, in accordance with Maṇḍana, triggering conditions). This move would have several consequences on the resulting system.
Extracting properties of the operator $\mathcal{O}$

As already explained in section 3.1, we decided to extract the axioms for $\mathcal{O}$ from the system $\text{bMDL}$. The extraction procedure is not trivial, since the axiomatic basis for $\text{bMDL}$ provided in Ciabattoni et al. 2015 includes no specific principle for $\mathcal{O}$ (namely, no principle in which $\mathcal{O}$ is the sole intensional operator). A general description of the procedure is the following: we first consider some relevant axioms and rules for dyadic $\mathcal{O}$ that are provable in $\text{bMDL}$ (under a given translation) and show that they are sufficient to axiomatize the set of all theorems of $\text{bMDL}$ in which the only intensional operator is $\mathcal{O}$, namely the dyadic $\mathcal{O}$-fragment of $\text{bMDL}$. Then we also illustrate how one can obtain the monadic $\mathcal{O}$-fragment of $\text{bMDL}$, by taking into account all theorems of $\text{bMDL}$ in which $\mathcal{O}$ is the only intensional operator and always has the formula $\top$ as its precondition.

**Theorem 3** The schemes $\neg(\mathcal{O}\phi \land \mathcal{O}\neg\phi)$, $\mathcal{O}\phi(\phi \land \psi) \rightarrow (\mathcal{O}\phi \land \mathcal{O}\psi)$, as well as the rules $\phi \equiv \psi\quad \mathcal{O}\phi \equiv \mathcal{O}\psi$ and $\frac{\theta \equiv \xi}{\mathcal{O}\phi \equiv \mathcal{O}\psi}$ hold in $\text{bMDL}$.

*Proof* The conjunction of (21) and (22) entails $\vdash_{\text{bMDL}} \Box(\phi \rightarrow \psi) \rightarrow (\mathcal{O}\phi \rightarrow (\mathcal{O}\psi \land \neg\mathcal{O}\neg\psi))$. By substitution, we get $\vdash_{\text{bMDL}} \Box(\phi \rightarrow \phi) \rightarrow (\mathcal{O}\phi \rightarrow (\mathcal{O}\phi \land \neg\mathcal{O}\neg\phi))$; since $\vdash_{\text{bMDL}} \Box(\phi \rightarrow \phi)$ (it is a normal operator in this system), then by Modus Ponens $\vdash_{\text{bMDL}} \mathcal{O}\phi \rightarrow \neg\mathcal{O}\neg\phi$, which is $\vdash_{\text{bMDL}} \neg(\mathcal{O}\phi \land \mathcal{O}\neg\phi)$. From $\vdash_{\text{bMDL}} \Box(\phi \rightarrow \psi) \rightarrow (\mathcal{O}\phi \rightarrow (\mathcal{O}\psi \land \neg\mathcal{O}\neg\psi))$ we can also get, by substitution, $\vdash_{\text{bMDL}} \Box(\Box(\phi \land \psi) \rightarrow \phi) \rightarrow (\mathcal{O}\phi \land \mathcal{O}\psi \land \neg\mathcal{O}\neg\phi))$ and, since $\vdash_{\text{bMDL}} \Box(\phi \land \psi) \rightarrow \phi$, then $\vdash_{\text{bMDL}} \mathcal{O}\phi(\phi \land \psi) \rightarrow (\mathcal{O}\phi \land \neg\mathcal{O}\neg\phi))$, which entails $\vdash_{\text{bMDL}} \mathcal{O}\phi(\phi \land \psi) \rightarrow \mathcal{O}\phi$. A parallel argument can be used to get $\vdash_{\text{bMDL}} \mathcal{O}\phi(\phi \land \psi) \rightarrow \mathcal{O}\phi$, hence $\vdash_{\text{bMDL}} \mathcal{O}\phi(\phi \land \psi) \rightarrow (\mathcal{O}\phi \land \mathcal{O}\phi \land \neg\mathcal{O}\neg\phi))$.

Assuming $\vdash_{\text{bMDL}} \Box(\phi \equiv \psi)$, we get $\vdash_{\text{bMDL}} \Box(\phi \equiv \psi)$, whence $\vdash_{\text{bMDL}} \Box(\phi \equiv \psi)$ and $\vdash_{\text{bMDL}} \Box(\phi \equiv \psi)$, from which one can infer, by (21), $\vdash_{\text{bMDL}} (\mathcal{O}\phi \rightarrow \mathcal{O}\psi) \land (\mathcal{O}\psi \rightarrow \mathcal{O}\phi)$, which is the same as $\vdash_{\text{bMDL}} \mathcal{O}\phi \equiv \mathcal{O}\psi$. Finally, by (23) one immediately gets $\vdash_{\text{bMDL}} (\Box(\theta \equiv \xi) \rightarrow \mathcal{O}\phi \equiv \mathcal{O}\phi)$. Assuming $\vdash_{\text{bMDL}} \theta \equiv \xi$, we obtain $\vdash_{\text{bMDL}} \Box(\theta \equiv \xi)$, hence $\vdash_{\text{bMDL}} \mathcal{O}\phi \rightarrow \mathcal{O}\xi$ and $\vdash_{\text{bMDL}} \mathcal{O}\xi \rightarrow \mathcal{O}\phi$, whence $\vdash_{\text{bMDL}} \mathcal{O}\phi \equiv \mathcal{O}\xi$.

Q.E.D.

We will hereafter call $\text{EMD}_d$ (i.e., dyadic EMD)\(^{37}\) the system obtained by extending a suitable axiomatic basis for the propositional calculus with the principles $\neg(\mathcal{O}\phi \land \mathcal{O}\neg\phi)$, $\mathcal{O}\phi(\phi \land \psi) \rightarrow (\mathcal{O}\phi \land \mathcal{O}\psi)$, $\phi \equiv \psi\quad \mathcal{O}\phi \equiv \mathcal{O}\psi$ and $\frac{\theta \equiv \xi}{\mathcal{O}\phi \equiv \mathcal{O}\psi}$. Let $\text{VAR} = \{p, q, r, \ldots\}$ be a set of propositional variables, $\mathcal{L}(\text{EMD}_d)$ denote the language of $\text{EMD}_d$. $\vdash_{\text{EMD}_d}$ in neighborhood frames of kind $\mathfrak{F} = (W, \{N_\theta : \theta \in \mathcal{L}(\text{EMD}_d)\})$ where $W$ is a set of worlds and, for any $\theta \in \mathcal{L}(\text{EMD}_d)$, $N_\theta : W \rightarrow \wp(\wp(W))$ is a neighborhood function. A model over such frames is a structure $\mathfrak{M} = (\mathfrak{F}, V)$ s.t. $V$ is a function mapping propositional variables to subsets of $W$. For any formula $\phi$ let $|\phi|^{\mathfrak{M}}$ be the truth-set of $\phi$ in $\mathfrak{M}$, namely $\{v \in W : \mathfrak{M}, v \models \phi\}$. Formulas are evaluated with reference to a world in a model and truth-conditions are as usual. In particular:

$- \mathfrak{M}, w \models \mathcal{O}\phi \iff |\phi|^{\mathfrak{M}} \in N_\theta(w)$.

Following Chellas 1980, we can claim that $\text{EMD}_d$ is complete w.r.t. the class of neighborhood frames having the following properties:

$- \text{if } X \in N(w) \text{ and } X \subseteq Y \subseteq W, \text{ then } Y \in N(w);$
$- \text{if } X \in N(w), \text{ then } X \notin N(w);$
$- \text{if } \vdash_{\text{EMD}_d} \theta \equiv \xi, \text{ then for all } X \in \wp(W), X \in N_\xi(w) \iff X \in N_\xi(w).$

The neighborhood frames defined in Ciabattoni et al. 2015 are structures of kind $\mathfrak{F} = (W, R, N)$, where $W$ is a set of worlds, $R \subseteq W \times W$ is an accessibility relation associated with $\Box$ and $N : W \rightarrow \wp(\wp(W) \times \wp(W))$ a neighborhood function associated with $\mathcal{O}$. Models over such frames and truth-conditions for formulas are defined as usual, except for:

$- \mathfrak{M}, w \models \mathcal{O}\phi \iff (|\phi|^{\mathfrak{M}} \cap R(w), |\theta|^{\mathfrak{M}} \cap R(w)) \in N(w).$

Notice that when $R$ is a universal relation (i.e., $R = W \times W$), this condition can be simplified to:

$- \mathfrak{M}, w \models \mathcal{O}\phi \iff (|\phi|^{\mathfrak{M}}, |\theta|^{\mathfrak{M}}) \in N(w).$

\(^{37}\) The name EMD is taken from Chellas 1980.
bMDL is complete w.r.t. the class of neighborhood frames having the following properties:

- \(R\) is transitive and reflexive;  
- if \((X, Y) \in N(w)\), then \(X, Y \subseteq R(w)\), where \(R(w) = \{v \in W : wRv\}\);  
- if \((X, Z) \in N(w)\) and \(X \subseteq Y \subseteq R(w)\), then \((Y, Z) \in N(w)\);  
- if \((X, Z) \in N(w)\), then \((X \cap R(w), Z) \notin N(w)\) (and this entails, given the previous properties, \((\emptyset, X) \notin N(w)\)).

We now show that any model over a frame for EMD\(_d\) can be converted into a model over a frame for bMDL. First, we need to specify a translation function \(t\) mapping formulas of \(\mathcal{L}(\text{EMD}_d)\) to formulas of \(\mathcal{L}(\text{bMDL})\). Consider the following clauses:

- for any propositional variable \(p \in \text{VAR}\), \(t(p) = p\); 
- \(t(\lnot \phi) = \lnot t(\phi)\);  
- \(t(\phi \rightarrow \psi) = t(\phi) \rightarrow t(\psi)\);  
- \(t(O_{\phi} \psi) = O_{t(\phi)} t(\psi)\).

Let \(\mathfrak{M} = (W, \{N_\theta : \theta \in \mathcal{L}(\text{EMD}_d)\}, V)\) be an arbitrary model over a frame for EMD\(_d\). We define a model \(\mathfrak{M}^* = (W^*, R^*, N^*, V^*)\) s.t. \(W^* = W\), \(R^* = W^* \times W^*\), \(V^* = V\) and, for any world \(w \in W^*\), \(N^*(w)\) is the set of all pairs \((X, Y)\) s.t.: 

- \(Y = \{t(\xi)|\xi|^{\mathfrak{M}^*}\}\) for some \(\xi \in \mathcal{L}(\text{EMD}_d)\);  
- \(X \supseteq Z\) for some \(Z \in \wp(W)\) s.t. \(Z \in N_\xi(w)\).

We show now that the translation function \(t\) makes the evaluation of any formula \(\phi \in \text{EMD}_d\) at any world \(w \in W\) invariant among \(\mathfrak{M}\) and \(\mathfrak{M}^*\).

**Theorem 4** For any world \(w \in W\) and any formula \(\phi \in \mathcal{L}(\text{EMD}_d)\), we have \(\mathfrak{M}, w \vdash \phi\) iff \(\mathfrak{M}^*, w \vdash t(\phi)\).

**Proof** We use an induction on the length of formulas in \(\mathcal{L}(\text{EMD}_d)\). Atomic and boolean cases are straightforward, given that \(V^* = V\). The only relevant case is \(\phi = O_{\psi} \phi\) and, by induction hypothesis, we can assume that \(||t(\psi)||^{\mathfrak{M}^*} = ||\psi||^{\mathfrak{M}^*}\). Let \(\mathfrak{M}, w \vdash O_{\psi} \phi\); then \(||\psi||^{\mathfrak{M}_w} \in N_\psi(w)\) and this entails, by definition of \(N^*(w)\), that \((||t(\psi)||^{\mathfrak{M}^*}, ||t(\phi)||^{\mathfrak{M}^*}) \in N^*(w)\), whence \(\mathfrak{M}^*, w \vdash O_{t(\phi)} t(\psi)\), i.e. \(\mathfrak{M}^*, w \vdash t(O_{\psi} \phi)\). Let \(\mathfrak{M}, w \not\vdash O_{\psi} \phi\); then \(||\psi||^{\mathfrak{M}_w} \notin N_\psi(w)\) and, since \(N_\psi(w)\) is closed under supersets (by definition of models for EMD\(_d\)), there is no \(Z \in \wp(W)\) s.t. \(Z \in N_\psi(w)\) and \(Z \subseteq ||\psi||^{\mathfrak{M}^*}\). From this one can infer \((||t(\psi)||^{\mathfrak{M}^*}, ||t(\phi)||^{\mathfrak{M}^*}) \notin N^*(w)\) and \(\mathfrak{M}^*, w \not\vdash O_{t(\phi)} t(\psi)\), i.e. \(\mathfrak{M}^*, w \not\vdash t(O_{\psi} \phi)\).

Q.E.D.

The most important consequence of Theorem 4 is that for any formula \(\phi \in \mathcal{L}(\text{EMD}_d)\), if \(\phi\) is falsified at some world \(w \in \mathfrak{M}\), then \(t(\phi)\) is falsified at the same world in \(\mathfrak{M}^*\).

Finally, it remains to show that \(\mathfrak{M}^*\) is a model (over a frame) for bMDL.

**Theorem 5** For any formula \(\phi \in \mathcal{L}(\text{bMDL})\), if \(\vdash_{\text{bMDL}} \phi\), then \(\vdash_{\text{EMD}_d} \phi\).

We use an induction on the length of derivations showing that all axioms of bMDL are valid in \(\mathfrak{M}^*\) and that rules of bMDL preserve validity in \(\mathfrak{M}^*\). First, notice that in \(\mathfrak{M}^*\) the accessibility relation \(R^*\) is universal, so all theorems of S4 are valid. Consider axiom (21), namely \((\Box(\phi \rightarrow \psi) \land O_{\psi} \phi) \rightarrow O_{\psi} \phi\). Assume that, for some \(w \in W^*\), we have \(\mathfrak{M}^*, w \vdash \Box(\phi \rightarrow \psi) \land O_{\psi} \phi\). This means that \(R^*(w) \subseteq ||\psi \rightarrow \phi||^{\mathfrak{M}^*}\), whence \(\mathfrak{M}^*, w \vdash O_{\psi} \phi\); then, we get \(R^*(w) \subseteq ||\psi \rightarrow \phi||^{\mathfrak{M}^*} \cap R^*(w)\). However, given that for any formula \(\chi\), \(|\chi||^{\mathfrak{M}^*} \cap R^*(w) = |\chi||^{\mathfrak{M}^*}\), then \(||\psi \rightarrow \phi||^{\mathfrak{M}^*} \subseteq ||\psi||^{\mathfrak{M}^*}\). We know that \(\mathfrak{M}^*, w \vdash O_{\psi} \phi\), so \(||\phi||^{\mathfrak{M}^*}, ||\theta||^{\mathfrak{M}^*} \in N^*(w)\) and this means (by definition of \(N^*(w)\)) that there is some \(\xi \in \mathcal{L}(\text{EMD}_d)\) and some \(Z \in \wp(W)\) s.t. \(Z \in N_\xi(w)\) and \(Z \subseteq ||\phi||^{\mathfrak{M}^*}\). From this one can infer \(\mathfrak{M}^*, w \vdash O_{\psi} \phi\).

Consider axiom (22), namely \(\Box(\phi \rightarrow \psi) \rightarrow \Box(O_{\psi} \phi \land O_{\phi} \psi)\). Assume that, for some \(w \in W^*\), we have \(\mathfrak{M}^*, w \vdash \Box(\phi \rightarrow \psi) \land O_{\phi} \psi\); then, we get \(R^*(w) \subseteq ||\psi \rightarrow \phi||^{\mathfrak{M}^*} \land ||\psi||^{\mathfrak{M}^*} \subseteq ||\phi||^{\mathfrak{M}^*}\). Furthermore, there is some \(\xi \in \mathcal{L}(\text{EMD}_d)\) and some \(Z \in \wp(W)\) s.t. \(Z \in N_\xi(w)\) and \(|\psi||^{\mathfrak{M}^*} \subseteq ||\phi||^{\mathfrak{M}^*}\). If it were the case that \(\mathfrak{M}^*, w \vdash O_{\phi} \psi\), then we would have, for some \(Q \in \wp(W)\), \(Q \in N_\xi(w)\) and \(Q \subseteq ||\phi||^{\mathfrak{M}^*}\). However, notice that \(Z \subseteq ||\phi||^{\mathfrak{M}^*}\); moreover, we know that \(Z \subseteq N_\xi(w)\). The definition of models for EMD\(_d\) tells us that \(N_\xi(w)\) is closed under supersets; thus (given that \(W = W^*\) and so \(||\phi||^{\mathfrak{M}^*} \subseteq ||\phi||^{\mathfrak{M}^*} \subseteq W^*\)) if it were the case that \(Q \subseteq N_\xi(w)\), one would get both \(||\phi||^{\mathfrak{M}^*} \subseteq N_\xi(w)\) and \(||\phi||^{\mathfrak{M}^*} \subseteq ||\phi||^{\mathfrak{M}^*}\). Consider axiom (23), namely \((\Box(\phi \rightarrow \theta) \land (\theta \rightarrow \psi)) \rightarrow O_{\phi} \psi\). Suppose \(\mathfrak{M}^*, w \vdash \Box(\phi \equiv \psi)\) for some \(w \in W^*\). Then, we get \(\mathfrak{M}^*, w \vdash O_{\phi} \psi\).
iff (||ϕ||^\mathcal{M}_{\Delta}, ||ψ||^\mathcal{M}_{\Delta}) \in N^*(w)$ iff (given that $||ψ||^\mathcal{M}_{\Delta} = ||θ||^\mathcal{M}_{\Delta}$ by assumption) $(||ϕ||^\mathcal{M}_{\Delta}, ||θ||^\mathcal{M}_{\Delta}) \in N^*(w)$ iff $\mathcal{M}_{\Delta}, w \models Oϕ; \text{ thus, } \mathcal{M}_{\Delta}, w \models Oϕ \equiv Oθϕ$, which is enough. The fact that necessitation and Modus Ponens (the only rules of bMDL) preserve validity in $\mathcal{M}_{\Delta}$ is straightforward.

Q.E.D.

Taken together, Theorem 4 and Theorem 5 entail that for any formula $ϕ \in \mathcal{L}(\text{EMD}_d)$, if $ϕ$ is falsifiable in some model for $\text{EMD}_d$, then there is a model for bMDL falsifying $ϕ$. Given the characterization results for $\text{EMD}_d$ and bMDL, this means that for any $ϕ \in \mathcal{L}(\text{EMD}_d)$, if $ϕ \models_{\text{EMD}_d} \text{EMD}_d t(ϕ)$. Since, by Theorem 3, we also have that if $ϕ \models_{\text{EMD}_d} \text{EMD}_d t(ϕ)$, then we can claim that $\text{EMD}_d$ is the dyadic $O$-fragment of bMDL under the translation function $t$.

As long as the monadic $O$-fragment of bMDL is concerned, let EMD be the monadic version of $\text{EMD}_d$; more precisely, $\text{EMD}$ is the system obtained by extending a suitable axiomatic basis for the propositional calculus with the principles $\neg(Oϕ \land O\negϕ), O(ϕ \land ψ) \rightarrow (Oϕ \land Oψ)$ and $Oϕ \equiv Oψ$. Now, we can define a translation function $f$ mapping formulas of $\mathcal{L}(\text{EMD})$, the language of $\text{EMD}$, to formulas of $\mathcal{L}(\text{EMD}_d)$, the language of $\text{EMD}_d$ (assuming, as usual, that the two languages are based on the same set of propositional variables $\text{VAR}$). Consider the following clauses (where $\top$ stands for an arbitrary tautology):

- for any propositional variable $p \in \text{VAR}$, $f(p) = p$;
- $f(\neg ϕ) = \neg f(ϕ)$;
- $f(ϕ \rightarrow ψ) = f(ϕ) \rightarrow f(ψ)$;
- $f(Oϕ) = O_t f(ϕ)$.

The intuition behind the last clause is that something is unconditionally obligatory iff it is obligatory under any condition which is trivially the case ($\top$): saying that one (unconditionally) ought to pay taxes is the same as saying that one ought to pay taxes given that it rains or it does not rain. Clearly, under the translation function $f$ $\text{EMD}$ corresponds to what we can call the $O_{\top}$-fragment of $\text{EMD}_d$, namely the system obtained by extending a suitable axiomatic basis of the propositional calculus with the principles $\neg(O_{\top}ϕ \land O_{\top}\negϕ), O_{\top}(ϕ \land ψ) \rightarrow (O_{\top}ϕ \land O_{\top}ψ)$, $\neg O_{\top}ϕ \equiv O_{\top}ψ$ and $O_{\top}ϕ \equiv O_{\top}ψ$. Indeed, using the translation function $f$ one can easily show that the first three principles are just the images under $f$ of the principles $\neg(Oϕ \land O\negϕ), O(ϕ \land ψ) \rightarrow (Oϕ \land Oψ)$ and $Oϕ \equiv Oψ$; furthermore, the fourth principle, namely $\neg O_{\top}ϕ \equiv O_{\top}ψ$, is equivalent to the image under $f$ of $Oϕ \equiv Oϕ$, which is a tautology. Theorems (3)-(5) and the translation function $t$ used above in this section guarantee that the $O_{\top}$-fragment of $\text{EMD}_d$ is also the $O_{\top}$-fragment of bMDL. Finally consider the translation function $g$ mapping formulas of $\text{EMD}$ to formulas of bMDL and resulting from the combination of $f$ and $t$: for any $ϕ \in \mathcal{L}(\text{EMD})$, $g(ϕ) = t(f(ϕ))$. We immediately get that $g$ satisfies the following clauses:

- for any propositional variable $p \in \text{VAR}$, $g(p) = p$;
- $g(\neg ϕ) = \neg g(ϕ)$;
- $g(ϕ \rightarrow ψ) = g(ϕ) \rightarrow g(ψ)$;
- $g(Oϕ) = O_t g(ϕ)$.

From this one can conclude that $\text{EMD}$ is the monadic $O$-fragment of bMDL under the translation function $g$.

On the interpretation of the operator $O$

The system bMDL, from which we extracted the principles for $O$, has among its theorems all instances of the schema $Oϕ \rightarrow Oϕ(ϕ \lor ψ)$. This schema is often taken to be puzzling for deontic reasoning, since it allows one to derive Ross’s paradox (see, e.g., Hilpinen and McNamear 2013): if $p$ stands for the proposition expressed by the sentence ‘Mark posts the letter’ and $q$ for the proposition expressed by the sentence ‘Mark burns the letter’, then one can read the formula $O_{\top}(p \lor q)$ as saying that Mark ought to post the letter or burn it. The problem is that in bMDL, as well as in its extensions treated in the present

\[\text{\footnotesize \cite{Hilpinen and McNamear 2013}}\]
article, one can infer $O_T(p \lor q)$ from $O_T p$, namely from the fact that Mark ought to post the letter (see also the monadic translation of the two formulas at issue under the function $g$ described above). At a first glance, this seems to be a relevant drawback of logical systems used to represent deontic modalities; however, a closer look at the intended semantics of $O$ reveals that the schema $O_\theta \phi \rightarrow O_\theta (\phi \lor \psi)$ can be interpreted in a plausible way.

Indeed, in this article the operator $O$ has to be read as ‘in every situation in which all strong Vedic duties are observed...’; such reading applies both to the monadic and to the dyadic case, as illustrated in section 3. Now, the truth of $O_\theta \phi$ in a situation $w$ means that $\phi$ describes a state-of-affairs holding in every situation where all strong Vedic duties applying to $w$ are observed and where the condition expressed by $\theta$ is fulfilled. Moreover, it is clearly appropriate to say that the state-of-affairs described by $\phi \lor \psi$ generalizes the state-of-affairs described by $\phi$; therefore, $\phi$ describes something which is sufficient to have $\phi \lor \psi$ true. From this one can conclude that $\phi \lor \psi$ cannot be false in any situation in which all strong Vedic duties applying to $w$ are observed and the condition $\theta$ is fulfilled. But this is exactly the same as saying that $O_\theta (\phi \lor \psi)$ is true at $w$.39

References


39 For a broader discussion and criticism of Ross’s paradox, the reader is referred to Castañeda 1981 and Äqvist 1987. Our argument is in line with the criticism provided in Cocchiarella 2015, which aims at defending the use of normal (hence, monotonic) systems of deontic logic.


Ollett, A. 2013, ‘What is Bhāvanā?’, *Journal of Indian Philosophy* 41(3), 221–262.


